Peer-to-Peer Systems

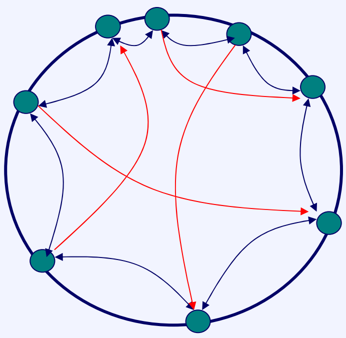
DHT examples, part 3
(Symphony, Viceroy, Distance Halving, Koorde)

Michael Welzl michael.welzl@uibk.ac.at

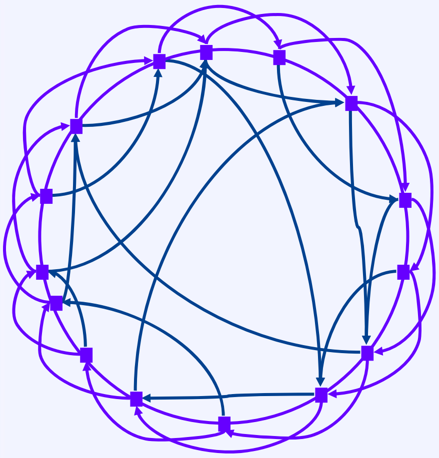
DPS NSG Team http://dps.uibk.ac.at/nsg
Institute of Computer Science
University of Innsbruck, Austria

Symphony

- Key idea: Distributed Hashing in a Small World
  - Start with Chord, but discard the strong requirements on the routing table (finger table); rely on small world (random links) to reach the destination
- Construction
  - Map the nodes and keys to the ring
  - Link every node with its successor and predecessor
  - Add k random links with probability proportional to 1/(d·log N), where d is the distance on the ring
  - Lookup time $O(\log^2 N)$
  - If k = log N lookup time O(log N)
  - Easy to insert and remove nodes (perform periodical refreshes for the links)
    - $O(\log^2 N)$ expected messages



Symphony /2



A typical Symphony network

Nodes arranged in a *unit circle* (perimeter = 1)

Arrival ⇒ Node chooses position along circle uniformly at random

Each node has *1 short link* (next node on circle) and *k long links*

Adaptation of Small World Idea: [Kleinberg00]
Long links chosen from a probability distribution function: p(x) = 1 / x log n where n = #nodes.

n ?

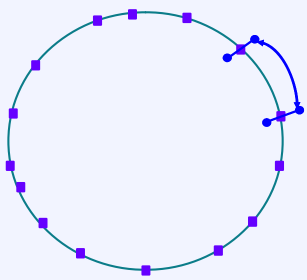
Simple greedy routing:
"Forward along that link that minimizes the absolute distance to the destination."

Average lookup latency = $O((\log^2 n) / k)$ hops

Fault Tolerance:
No backups for long links! Only short links are fortified for fault tolerance.

Symphony /3

- Key problem: network size estimation
  - Based on family of harmonic functions (as PDF), hence the name



x = Length of arc
1/x = Estimate of n
p(x)= 1 / (x log n)

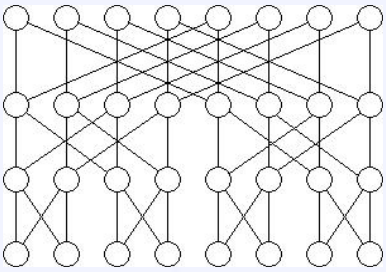
- Symphony optimizations:
  - Bi-directional Routing
    - Exploit both outgoing and incoming links!
    - Route to the neighbor that minimizes absolute distance to destination
    - Reduces avg latency by 25-30%
  - 1-Lookahead
    - List of neighbor's neighbors; reduces avg. latency by 40%

Viceroy

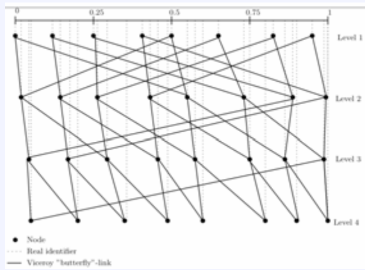
Thanks to google :-)

- is a butterfly

- Butterfly = well known network topology with some desirable properties
  - Small degree (4) and small (proven to be close to optimal) diameter
  - Logarithmic path length between any two nodes
  - Simple routing, no bottlenecks, high resilience

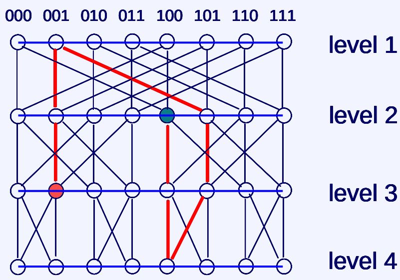


Theory



Ideal case in Viceroy

Routing in a butterfly network

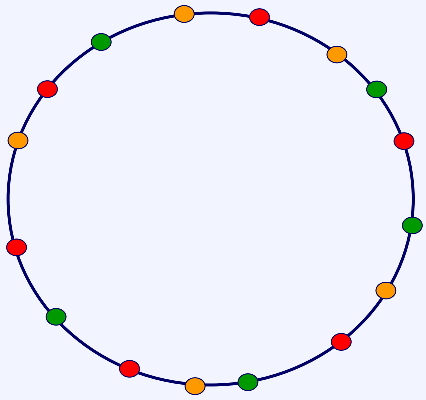


- That's a bit complicated and inefficient

- Hence, in Viceroy, nodes are also connected within each level

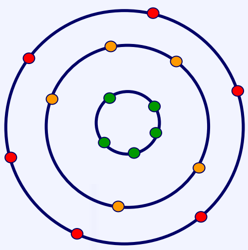
Viceroy network

- Arrange nodes and keys on a ring
  - like in Chord
- Assign to each node a level value
  - chosen uniformly from the set {1,…,log n}
  - estimate n by taking the inverse of the distance of the node with its successor
  - easy to update



Viceroy network /2

- Create a ring of nodes within the same level
  - In addition to a "general" ring of all nodes
- Each node x at level i has two downward links to level i+1
  - a left link to the first node of level i+1 after position x on the ring
  - a right link to the first node of level i+1 after pos. $x + (½)^i$



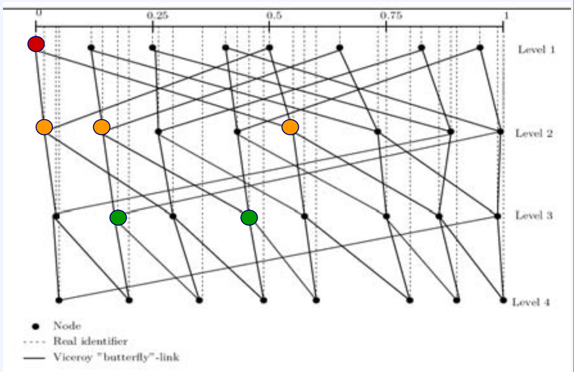
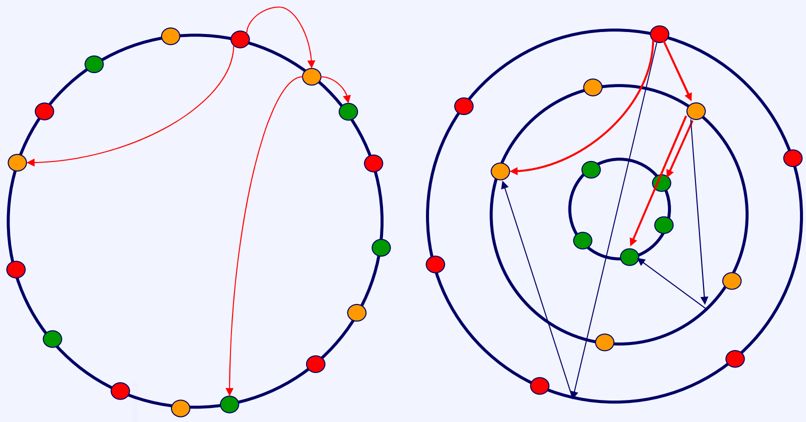


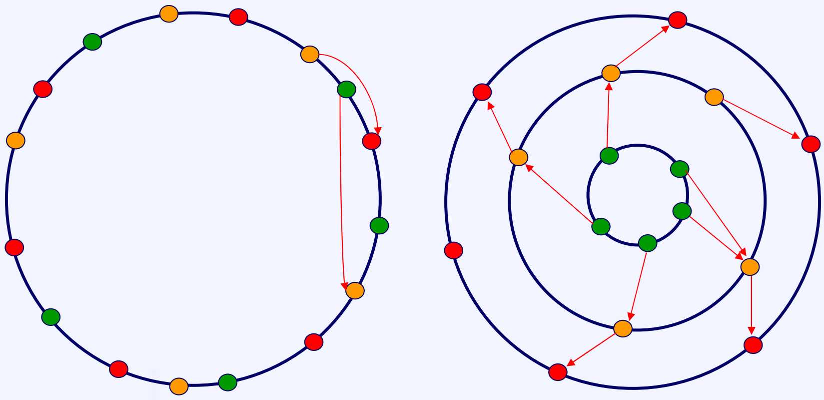
Figure 1: An ideal Viceroy network. Up and ring links are omitted for simplicity.

Downward links



Upward links

- Each node x at level i has an upward link to the next node on the ring at level i-1



Viceroy: Joining

1. Insert peer at random position of the general ring
2. Estimate log n by looking at the distance between a node and its successor
3. Randomly pick level i (uniformly distributed between 1 and log n)
4. Find position in ViceRoy network via lookup starting at the ring neighbor
5. Insert peer into the ViceoRoy network level by
   - Inserting peer in ring i of the network
   - Finding the...
     - Successor of (i,x)
     - Successor of (i+1,x)
     - Successor of (i+1,x+2^i)
     - Predecessor of (i-1, x)
     - Predecessor of (i-1, x-2^i)
   - ...starting at the edges connected to the neighbor in ring i

- Complexity
  - Lookup time (O(log n)) +
  - Finding the successor / predecessor (O(log n))

Viceroy: Searching

- Peer (i,x) gets search request for (j,y)

```
IF i=j und |x-y| ≤ (log n)²/n THEN
    Forward search request to neighbor of ring i
ELSE
    IF y is to the right of x+2^i THEN
        Forward request to successor of (i+1,x+2^i)
    ELSE
        Forward request to Z = successor of (i+1,x)
        IF successor Z is to the right of x THEN
            Search a node (i+1, p) with p<x on the ring (i+1), starting at Z
        FI
    FI
FI
```

- With a high probability, this takes time (and messages) of O(log n)

Viceroy conclusion

- First Peer-to-Peer network with constant in- and outdegree
  - Outdegree 8, Indegree should be constant
    - additional "multiple choice" mechanism was added to insertion procedure to truly make it constant
(not included on previous slide about insertion for simplicity)

- ...but:
  - Multiple ring structure quite complex
  - Multiple choice method causes O(log2n) insertion complexity
  - As we will see, there are easier networks with similar properties

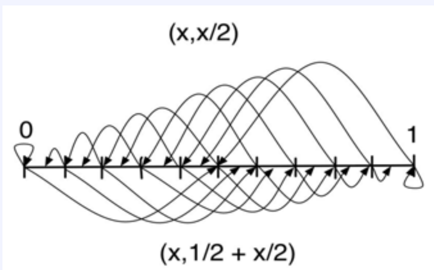
Distance Halving

- Published by Moni Naor and Udi Wieder in 2003
  - Moni Naor is also a coauthor of the Viceroy paper :-)

- Based on continuous graphs
  - Infinite graphs with continuous node and edge set

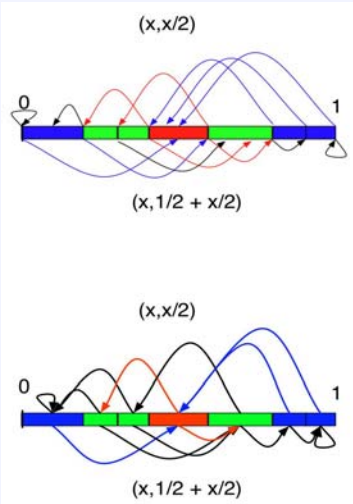
- In Distance Halving:
  - Nodes: $x \in [0,1)$
  - Edges:
    - Left-edges: (x,x/2)
    - Right-edges: (x,1/2+x/2)
    - and edges back:
      - (x/2,x)
      - (1/2+x/2,x)

    - Note that distance halves with every step $\Rightarrow$ hence the name :-)



Discretization

- Consider fixed number of discrete intervals in the continuous space
  - formed by successive halving
- Insert an edge between intervals A and B If there are x ∈ A and y ∈ B such that (x,y) is an edge of the continuous graph
- Possible (simple) implementation: peers pick a random position in [0,1)
  - They are responsible for data from their position to their successor
- Neighboring intervals are also bidirectionally connected (ring)



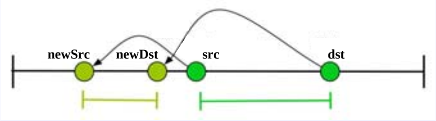
Multiple choice principle

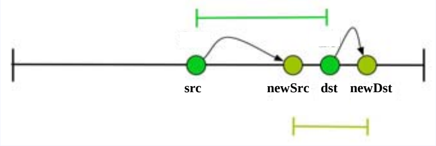
- Goal, as in Viceroy: constant degree
  - Emerges if ratio between largest and smallest interval is constant
  - With a high probability, largest interval = 2/n, smallest interval = 1/(2n) ⇒ constant degree
  - ... and logarithmic diameter
- Degree of 4 can be achieved via multiple choice principle for joining (goal: evenly spread nodes across range):
  - Send c log n queries to randomly chosen intervals
  - Select largest interval and halve it
  - Update ring edges
  - Update left- and right-edges
- Time and number of messages for inserting peers: $O(\log^2 n)$

Routing

- Distance is halved with each step
  - O(log n) hops and messages

- Example algorithm:
  Left-Routing (src, dst)
  - IF dst is in neighbor interval
    - Forward query to dst
  - ELSE
    - newSrc = left-edge(src);
    - newDst = left-edge(dst);
    - Send message from src to newSrc;
    - Left-Routing(newSrc, newDst);
    - Send message from newDst to dst;





- Note: this only uses left-edges
  - Could also be done with right-edges only

Routing and conclusion

- Left- and right-edges can be combined using an arbitrary strategy (alternate, random, ..)
  - Congestion (number of packets transmitted by each peer) is O(log n) in the worst case (when every peer sends a request)
  - Proof based on similar proof for hypercube; same result can also be shown for Viceroy

- Conclusion: simple and efficient structure
  - degree O(1), diameter O(log n), lookup O(log n), join O($\log^2$n), load balancing

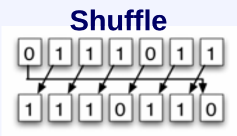
- Principle of discretizing continuous graphs also used in other DHTs
  - But this is the first time the problem was explicitly formulated like this

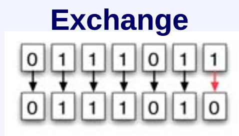
Enhancing Chord: degree or diameter?

- Chord: degree O(log n), diameter O(log n)
  - Making these smaller is desirable
- Question 1: can we get a smaller diameter with degree g=O(log n)?
  - Distance 1: at most g nodes
  - Distance 2: at most g^2 nodes
  - ⇒ thus, distance d: g^d nodes
- Hence: $(\log n)^d = n$
- It follows that: $d = \frac{\log n}{\log \log n}$
- Therefore, only minor improvement of diameter possible

Koorde

- Karger, Kasshoek (2003)
- Goal: maintain Chord's O(log n) diameter make in- and outdegree = 2
  - This can be done with a binary tree, a butterfly net, a DeBruijn graph…
- Foundation: operations on binary string S of length m
  - Shuffle:
    - shuffle(s_1, s_2, s_3,…, s_m) = (s_2,s_3,…, s_m,s_1)
  - Exchange:
    - exchange(s_1, s_2, s_3,…, s_m) = (s_1, s_2, s_3,…, $\neg s_m$)
  - Shuffle-Exchange:
    - SE(S) = exchange(shuffle(S)) = (s_2,s_3,…, s_m, $\neg$ s_1)





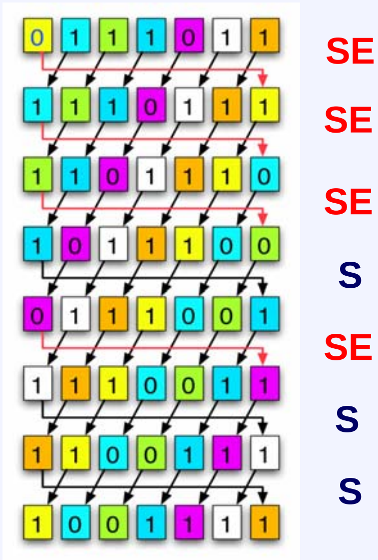
Shuffle and exchange

- Any string A can be turned into any string B by applying Shuffle and Shuffle-exchange operations m times

Example:

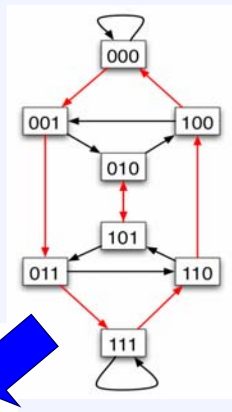
From	0	1	1	1	0	1	1
to	1	0	0	1	1	1	1
via	SE	SE	SE	S	SE	S	S

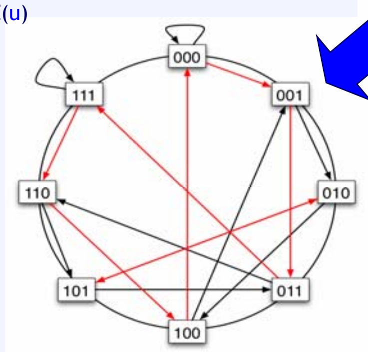
operations



The DeBruijn Graph

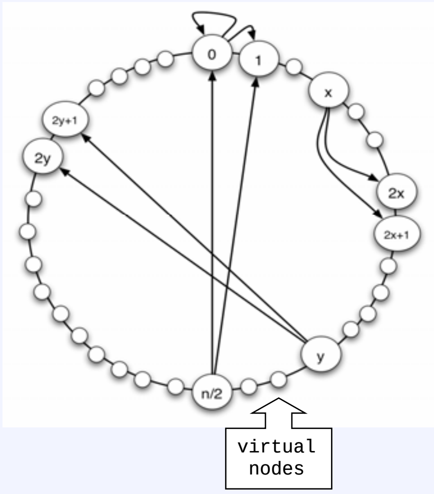
- A DeBruijn graph consists of $n=2^m$ nodes
  - represented as m-digit binary numbers
- Every node has two outgoing edges
  - 1. edge points from u to shuffle(u)
  - 2. edge points from u to SE(u)
- DeBruijn-Graph has degree 2 and $m=\log_2 n$ diameter
- Koorde = Ring + DeBruijn-Graph
  - Ring of 2^m nodes + DeBruijn edges





Koorde = Ring + DeBruijn Graph

- Edges
  - shuffle($s_1, s_2, \ldots, s_m$) = ($s_2, \ldots, s_m, s_1$)
    $\Rightarrow$ shuffle (x) = (x div 2^m)+ (2x) mod 2^m
  - SE(S) = ($s_2, s_3, \ldots, s_m, \neg s_1$)
    $\Rightarrow$ SE(x) = 1-(x div 2^m)+ (2x) mod 2^m
  - (x div 2^m) can be either 0 or 1
    $\Rightarrow$ successors of x are 2x mod 2^m and 2x+1 mod 2^m
- Exactly 2^m nodes unlikely in a P2P network
  - Choose large m (typically 128 or 160) $\Rightarrow$ more DeBruijn nodes than peers
- Unoccupied DeBruijn nodes become "virtual nodes"
  - Every peer manages all DeBruijn nodes between itself and successor
  - Only necessary for incoming edges

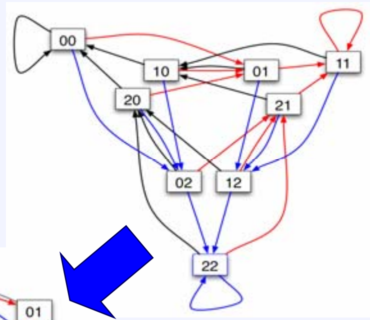


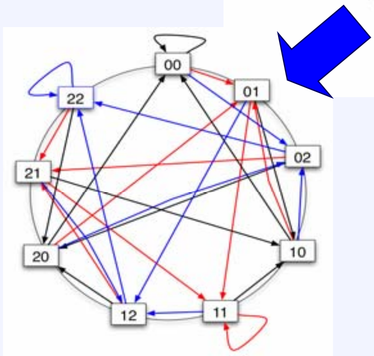
Koorde properties

- Per definition, four edges per node
- With high probability
  - at most O(log n) incoming edges per node
    - Reason:
      - distance to next peer is at most c (log n)/2^m (with high probability)
      - this is the max. interval from which peers can point to a peer (and its virtual nodes)
      - Within this interval, there are at most O(log n) peers with high probability
  - Diameter = O(log n)
  - Routing requires O(log n) messages
- But low coherence of Koorde graph

K-degree DeBruijn Graph

- Consider alphabet over k letters, e.g. k = 3
- Each k-DeBruijn-node x has successors
  - (kx mod k^m), (kx +1 mod k^m), (kx+2 mod k^m), ... , (kx+k-1 mod k^m)
- Diameter becomes (log m)/(log k)
- Coherence grows with k





References / acknowledgments

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