5mm.

INF1100 Lectures, Chapter 4: Input Data and Error Handling

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Programs until now hardcode input data

- Example:

```
v0 = 5
g = 9.81
t = 0.6
y = v0*t - 0.5*g*t**2
print y
```

- Input data are explicitly set (hardcoded)
- To change input data, we need to *edit* the program
- This is considered bad programming
  (because editing programs may easily introduce errors!)
- Rule: read input from user – do not edit a correct program

How do professional programs get their input?

- Consider a web browser: how do you specify a web address? How do you change the font?
- You don't need to go into the program and edit it...

How can we specify input data in programs?

- Until now: hardcoded initialization of variables
- Ch. 3: Ask the user questions and read answers
- Ch. 3: Read command-line arguments

```
Unix/DOS> python myprog.py arg1 arg2 arg3 ...
Unix/DOS> rm -i -r temp projects univ
```

  Unix programs (`rm`, `ls`, `cp`, ...) make heavy use of command-line arguments, see e.g. `man ls`)
- Beyond scope: Read data from special fields in windows on the screen
  (most Windows, Mac and Linux programs work this way)
- Ch. 7: Read data from file

Getting input from questions and anwsers

- Sample program:

```
C = 21
F = (9.0/5)*C + 32
print F
```

- Idea: let the program ask the user a question "C=?", read the user's answer, assign that answer to the variable C
- This is easy:

```
C = raw_input('C=? ')   # C becomes a string
C = float(C)
F = (9./5)*C + 32
print F
```

- Testing:

```
Unix/DOS> python c2f_qa.py
C=? 21
69.8
```


Print the n first even numbers

- Read n from the keyboard:

```
n = int(raw_input('n=? '))

for i in range(2, 2*n+1, 2):
    print i

# or:
print range(2, 2*n+1, 2)

# or:
for i in range(1, n+1):
    print 2*i
```


The magic eval function

- `eval(s)` evaluates a string object `s` as if the string had been written directly into the program
- Example `r = eval('1+1')` is the same as

```
r = 1+1
```

- Some other examples:

```
>>> r = eval('1+2')
>>> r
3
>>> type(r)
<type 'int'>

>>> r = eval('[1, 6, 7.5]')
>>> r
[1, 6, 7.5]
>>> type(r)
<type 'list'>
```


Be careful with eval and string values

- Task: write `r = eval(...)` that is equivalent to

```
r = 'math programming'
```

- Must use quotes to indicate that `'math programming'` is string, plus extra quotes:

```
r = eval("'math programming'")
# or
r = eval('"math programming"')
```

- What if we forget the extra quotes?

```
r = eval('math programming')
```

  is the same as

```
r = math programming
```

  but then Python thinks `math` and `programming` are two variables...combined with wrong syntax

A little program can do much

- This program adds two input variables:

```
i1 = eval(raw_input('operand 1: '))
i2 = eval(raw_input('operand 2: '))
r = i1 + i2
print '%s + %s becomes %s
with value %s' %       (type(i1), type(i2), type(r), r)
```

- We can add integer and float:

```
Unix/DOS> python add_input.py
operand 1: 1
operand 2: 3.0
<type 'int'> + <type 'float'> becomes <type 'float'>
with value 4
```

- We can add two lists:

```
Unix/DOS> python add_input.py
operand 1: [1,2]
operand 2: [-1,0,1]
<type 'list'> + <type 'list'> becomes <type 'list'>
with value [1, 2, -1, 0, 1]
```

- Note: `r = i1 + i2` becomes the same as

```
r = [1,2] + [-1,0,1]
```


This great flexibility also quickly breaks programs...

```
 Unix/DOS> python add_input.py
operand 1: (1,2)
operand 2: [3,4]
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: can only concatenate tuple (not "list") to tuple

Unix/DOS> python add_input.py
operand 1: one
Traceback (most recent call last):
  File "add_input.py", line 1, in <module>
    i1 = eval(raw_input('operand 1: '))
  File "<string>", line 1, in <module>
NameError: name 'one' is not defined

Unix/DOS> python add_input.py
operand 1: 4
operand 2: 'Hello, World!'
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: unsupported operand type(s) for +: 'int' and 'str'
```


A similar magic function: exec

- `eval(s)` evaluates an *expression* s
- `eval('r = 1+1')` is illegal because this is a statement, not only an expression (assignment statement: variable = expression)
- ...but we can use `exec` for complete statements:

```
statement = 'r = 1+1'   # store statement in a string
exec(statement)
print r
```

will print 2

- For longer code we can use multi-line strings:

```
somecode = '''
def f(t):
    term1 = exp(-a*t)*sin(w1*x)
    term2 = 2*sin(w2*x)
    return term1 + term2
'''
exec(somecode)  # execute the string as Python code
```


What can exec be used for?

- Build code at run-time, e.g., a function:

```
formula = raw_input('Write a formula involving x: ')
code = """
def f(x):
    return %s
""" % formula
exec(code)

x = 0
while x is not None:
    x = eval(raw_input('Give x (None to quit): '))
    if x is not None:
        y = f(x)
        print 'f(%g)=%g' % (x, y)
```

- While the program is running, the user types a formula, which becomes a function, the user gives `x` values until the answer is `None`, and the program evaluates the function `f(x)`
- Note: the programmer knows nothing about `f(x)`!

StringFunction: string formulas → functions

- It is common for programs to read formulas and turn them into functions so we have made a special tool for this purpose:

```
>>> from scitools.StringFunction import StringFunction
>>> formula = 'exp(x)*sin(x)'
>>> f = StringFunction(formula)
>>> f(0)
0.0
>>> f(pi)
2.8338239229952166e-15
```

- The function can have parameters: $g(t) = Ae^{-at}\sin(\omega x)$

```
g = StringFunction('A*exp(-a*t)*sin(omega*x)',
    independent_variable='t', A=1, a=0.1, omega=pi, x=5)
print g(1.2)
g.set_parameters(A=2, x=10)
print g(1.2)
```


Reading from the command line

- Consider again our Celsius-Fahrenheit program:

```
C = 21; F = (9.0/5)*C + 32; print F
```

- Now we want to provide `C` as a command-line argument after the name of the program when we run the program:

```
Unix/DOS> python c2f_cml_v1.py 21
69.8
```

- Command-line arguments = "words" after the program name
- The list `sys.argv` holds the command-line arguments:

```
import sys
print 'program name:', sys.argv[0]
print '1st command-line argument:', sys.argv[1] # string
print '2nd command-line argument:', sys.argv[2] # string
print '3rd command-line argument:', sys.argv[3] # string
etc.
```

- The Celsius-Fahrenheit conversion program:

```
import sys
C = float(sys.argv[1])
F = 9.0*C/5 + 32
print F
```


Example on reading 3 parameters from the command line

- Compute the current location of an object,

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

when s_0 (initial location), v_0 (initial velocity), a (constant acceleration) and t (time) are given on the command line

- How far away is the object at $t = 3$ s, if it started at $s = 1$ m at $t = 0$ with a velocity $v_0 = 1$ m/s and has undergone a constant acceleration of 0.5 m/s^2?

```
Unix/DOS> python location_cml.py 1 1 0.5 3
6.25
```

- Program:

```
import sys
s0 = float(sys.argv[1])
v0 = float(sys.argv[2])
a  = float(sys.argv[3])
t  = float(sys.argv[4])
s  = s0 + v0*t + 0.5*a*t*t
print s
```


How are command-line arguments separated?

- Command-line arguments are separated by blanks – use quotes to override this rule!
- Let us make a program for printing the command-line args.:

```
import sys; print sys.argv[1:]
```

- Demonstrations:

```
Unix/DOS> python print_cml.py 21 string with blanks 1.3
['21', 'string', 'with', 'blanks', '1.3']

Unix/DOS> python print_cml.py 21 "string with blanks" 1.3
['21', 'string with blanks', '1.3']
```

- Note that all list elements are surrounded by quotes, showing that command-line arguments are strings

Command-line arguments with options

- Many programs, especially on Unix systems, take a set of command-line arguments of the form `-option value`

```
Unix/DOS> python location.py --v0 1 --t 3 --s0 1 --a 0.5
```

- Can use the module `getopt` to help reading the data:

```
s0 = 0; v0 = 0; a = t = 1   # default values
import getopt, sys
options, args = getopt.getopt(sys.argv[1:], '',
                ['t=', 's0=', 'v0=', 'a='])

# options is a list of 2-tuples (option,value) of the
# option-value pairs given on the command line, e.g.,
# [('--v0', 1.5), ('--t', 0.1), ('--a', 3)]

for option, value in options:
    if option == '--t':
        t = float(value)
    elif option == '--a':
        a = float(value)
    elif option == '--v0':
        v0 = float(value)
    elif option == '--s0':
        s0 = float(value)
```


Multiple versions of command-line args (long and short)

- We can allow both long and shorter options, e.g. `-t` and `-time`, and `-a` and `-acceleration`

```
options, args = getopt.getopt(sys.argv[1:], '',
  ['v0=', 'initial_velocity=', 't=', 'time=',
   's0=', 'initial_velocity=', 'a=', 'acceleration='])

for option, value in options:
    if option in ('--t', '--time'):
        t = float(value)
    elif option in ('--a', '--acceleration'):
        a = float(value)
    elif option in ('--v0', '--initial_velocity'):
        v0 = float(value)
    elif option in ('--s0', '--initial_position'):
        s0 = float(value)
```


Summary of `-option value` pairs

- Advantage of `-option value` pairs:
  - can give options and values in arbitrary sequence
  - can skip option if default value is ok
- Command-line arguments that we read as `sys.argv[1]`, `sys.argv[2]`, etc. are like positional arguments to functions: the right sequence of data is essential!
- `-option value` pairs are like keyword arguments – the sequence is arbitrary and all options have a default value

Handling errors in input

- A user can easily use our program in a wrong way, e.g.,

```
Unix/DOS> python c2f_cml_v1.py
Traceback (most recent call last):
  File "c2f_cml_v1.py", line 2, in ?
    C = float(sys.argv[1])
IndexError: list index out of range
```

(the user forgot to provide a command-line argument...)

- How can *we* take control, explain what was wrong with the input, and stop the program without strange Python error messages?

```
if len(sys.argv) < 2:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)  # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

- Execution:

```
Unix/DOS> python c2f_cml_v2.py
You failed to provide a command-line arg.!
```


Exceptions instead of if tests

- Rather than test "if something is wrong, recover from error, else do what we indended to do", it is common in Python (and many other languages) to *try* to do what we indend to, and if it fails, we recover from the error
- This principle makes use of a `try-except` block:

```
try:
    <statements we indend to do>
except:
    <statements for handling errors>
```

- If something goes wrong in the `try` block, Python raises an *exception* and the execution jumps immediately to the `except` block
- Let's see it in an example!

Celsius-Fahrenheit conversion with try-except

- Try to read C from the command-line, if it fails, tell the user and abort execution:

```
import sys
try:
    C = float(sys.argv[1])
except:
    print 'You failed to provide a command-line arg.!'
    sys.exit(1)  # abort
F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

- Execution:

```
Unix/DOS> python c2f_cml_v3.py
You failed to provide a command-line arg.!

Unix/DOS> python c2f_cml_v4.py 21C
You failed to provide a command-line arg.!
```


Testing for a specific exception

- In

```
try:
    <statements>
except:
    <statements>
```

we jump to the `except` block for *any* exception raised when executing the `try` block

- It is good programming style to test for specific exceptions:

```
try:
    C = float(sys.argv[1])
except IndexError:
    ...
```

- If we have an index out of bounds in `sys.argv`, an `IndexError` exception is raised, and we jump to the `except` block
- If any other exception arises, Python aborts the execution:

```
Unix/DOS>> python c2f_cml_tmp.py 21C
Traceback (most recent call last):
  File "tmp.py", line 3, in <module>
    C = float(sys.argv[1])
ValueError: invalid literal for float(): 21C
```


Branching into different except blocks

- We can test for different exceptions:

```
import sys
try:
    C = float(sys.argv[1])
except IndexError:
    print 'No command-line argument for C!'
    sys.exit(1)  # abort execution
except ValueError:
    print 'Celsius degrees must be a pure number, '\
          'not "%s"' % sys.argv[1]
    sys.exit(1)

F = 9.0*C/5 + 32
print '%gC is %.1fF' % (C, F)
```

- Execution:

```
Unix/DOS> python c2f_cml_v3.py
No command-line argument for C!

Unix/DOS> python c2f_cml_v3.py 21C
Celsius degrees must be a pure number, not "21C"
```

The programmer can raise exceptions (part 1)

- Instead of just letting Python raise exceptions, we can raise our own and tailor the message to the problem at hand
- We provide two examples on this:
  - catching an exception, but raising a new one with an improved (tailored) error message
  - rasing an exception because of wrong input data
- Example:

```
def read_C():
    try:
        C = float(sys.argv[1])
    except IndexError:
        raise IndexError\
        ('Celsius degrees must be supplied on the command line')
    except ValueError:
        raise ValueError\
        ('Celsius degrees must be a pure number, '\
         'not "%s"' % sys.argv[1])
    # C is read correctly as a number, but can have wrong value:
    if C < -273.15:
        raise ValueError('C=%g is a non-physical value!' % C)
    return C
```


The programmer can raise exceptions (part 2)

- Calling the function in the main program:

```
try:
    C = read_C()
except (IndexError, ValueError), e:
    # print exception message and stop the program
    print e
    sys.exit(1)
```

- Examples on running the program:

```
Unix/DOS> c2f_cml.py
Celsius degrees must be supplied on the command line

Unix/DOS> c2f_cml.py 21C
Celsius degrees must be a pure number, not "21C"

Unix/DOS> c2f_cml.py -500
C=-500 is a non-physical value!

Unix/DOS> c2f_cml.py 21
21C is 69.8F
```


Graphical user interfaces

- Most programs today fetch input data from *graphical user interfaces* (GUI), consisting of windows and graphical elements on the screen: buttons, menus, text fields, etc.
- Why don't we learn to make such type of programs?
  - GUI demands much extra complicated programming
  - GUI is an advantage for novice users
  - Experienced users often prefer command-line input (it's much quicker and can be automated)
  - The authors of a program are very experienced users...
  - Programs with command-line or file input can easily be combined with each other, this is difficult with GUI-based programs
- Assertion: command-line input will probably fill all your needs in university courses
- But let's have a look at GUI programming!

A graphical Celsius-Fahrenheit conversion program

- The Celsius degrees can be filled in as a number in a field
- Clicking the "is" button computes the corresponding Fahrenheit temperature

The GUI code


```
from Tkinter import *
root = Tk()
C_entry = Entry(root, width=4)
C_entry.pack(side='left')
Cunit_label = Label(root, text='Celsius')
Cunit_label.pack(side='left')

def compute():
    C = float(C_entry.get())
    F = (9./5)*C + 32
    F_label.configure(text='%g' % F)

compute = Button(root, text=' is ', command=compute)
compute.pack(side='left', padx=4)

F_label = Label(root, width=4)
F_label.pack(side='left')
Funit_label = Label(root, text='Fahrenheit')
Funit_label.pack(side='left')

root.mainloop()
```


Making your own modules

- We have used modules:

```
from math import log
r = log(6)   # call log function in math module

import sys
x = eval(sys.argv[1])  # access list argv in sys module
```

- A module is a collection of useful data and functions (later also classes)
- Functions in a module can be reused in many different programs
- If you have some general functions that can be handy in more than one program, you should consider making a module containing these functions, and then you can do

```
import mymodule
r = mymodule.my_resuable_function(arg1, arg2, arg3)
```

- It's very easy to make modules: just collect the functions you want in a file, and that's a module!

Example on making a module (part 1)

Here are formulas for computing with interest rates:

$$
\begin{aligned}
A &= A_0\left(1+\frac{p}{360\cdot 100}\right)^n \\
A_0 &= A\left(1+\frac{p}{360\cdot 100}\right)^{-n} \\
n &= \frac{\ln\frac{A}{A_0}}{\ln\left(1+\frac{p}{360\cdot 100}\right)} \\
p &= 360\cdot 100\left(\left(\frac{A}{A_0}\right)^{1/n}-1\right)
\end{aligned}
$$

A_0: initial amount, p: percentage, n: days, A: final amount
We want to make a module with these four functions

Example on making a module (part 2)

Python code for the four functions:

```
from math import log as ln

def present_amount(A0, p, n):
    return A0*(1 + p/(360.0*100))**n

def initial_amount(A, p, n):
    return A*(1 + p/(360.0*100))**(-n)

def days(A0, A, p):
    return ln(A/A0)/ln(1 + p/(360.0*100))

def annual_rate(A0, A, n):
    return 360*100*((A/A0)**(1.0/n) - 1)
```

Making the module

- Collect the 4 functions in a file `interest.py`
- Now `interest.py` is actually a module `interest` (!)
- Here is a program that applies this module:

```
# How long does it take to double an amount of money?

from interest import days
A0 = 1; A = 2; p = 5
n = days(A0, 2, p)
years = n/365.0
print 'Money has doubled after %.1f years' % years
```


Adding a test block in a module file

- Module files can have an if test at the end containing a *test block* for testing or demonstrating the module
- The test block is not executed when the file is imported as a module in another program
- The test block is executed *only* when the file is run as a program
- In our example we can have a verification as test block:

```
if __name__ == '__main__':
    A = 2.2133983053266699
    A0 = 2.0
    p = 5
    n = 730
    print 'A=%g (%g)
A0=%g (%.1f)
n=%d (%d)
p=%g (%.1f)' % \
          (present_amount(A0, p, n), A,
           initial_amount(A, p, n), A0,
           days(A0, A, p), n,
           annual_rate(A0, A, n), p)
```


Test blocks are often collected in functions

- Alternative code organization:

```
def _verify():
    # compatible values:
    A = 2.2133983053266699; A0 = 2.0; p = 5; n = 730
    # given three of these, compute the remaining one
    # and compare with the correct value (in parenthesis):
    A_computed = present_amount(A0, p, n)
    A0_computed = initial_amount(A, p, n)
    n_computed = days(A0, A, p)
    p_computed = annual_rate(A0, A, n)
    print 'A=%g (%g)
A0=%g (%.1f)
n=%d (%d)
p=%g (%.1f)' % \
          (A_computed, A, A0_computed, A0,
           n_computed, n, p_computed, p)

if __name__ == '__main__':
    if len(sys.argv) == 2 and sys.argv[1] == 'verify':
        _verify()
```


How can Python find our new module?

- If the module is in the same folder as the main program, everything is simple and ok
- "Home-made" modules are normally collected in a common folder, say `/Users/hpl/lib/python/mymods`
- In that case Python must be notified that our module is in that folder

Different techniques:

- add folder to `PYTHONPATH` in `.bashrc`:

```
export PYTHONPATH=$PYTHONPATH:/Users/hpl/lib/python/mymods
```

- add folder to `sys.path` in the program:

```
sys.path.insert(0, '/Users/hpl/lib/python/mymods')
```

- add the module file in a directory that Python already know contains useful libraries

Making a useful program of our new module

- Give three parameters on the command line
- Let the program (test block) compute the fourth

```
interest.py A0=2 A=1 n=1095    # compute p
```

- Use `exec` to execute the statements on the command line, find the missing parameter and call the appropriate function

```
init_code = ''
for statement in sys.argv[1:]:
    init_code += statement + '\n'
exec(init_code)   # initialize input parameters

if 'A=' not in init_code:
    print 'A =', present_amount(A0, p, n)
elif 'A0=' not in init_code:
    print 'A0 =', initial_amount(A, p, n)
elif 'n=' not in init_code:
    print 'n =', days(A0, A , p)
elif 'p=' not in init_code:
    print 'p =', annual_rate(A0, A, n)
```

- Study the book for understanding all details of this module

Summary of reading from the keyboard and command line

- Question and answer input:

```
var = raw_input('Give value: ')   # var is string!

# if var needs to be a number:
var = float(var)
# or in general:
var = eval(var)
```

- Command-line input:

```
import sys
parameter1 = eval(sys.argv[1])
parameter3 = sys.argv[3]          # string is ok
parameter2 = eval(sys.argv[2])
```

- Recall: `sys.argv[0]` is the program name

Summary of reading command-line arguments with getopt

- `-option value` pairs with the aid of `getopt`:

```
import getopt
options, args = getopt.getopt(sys.argv[1:], '',
      ['parameter1=', 'parameter2=', 'parameter3=',
       'p1=', 'p2=', 'p3=']  # shorter forms

# set default values:
parameter1 = ...
parameter2 = ...
parameter3 = ...

from scitools.misc import str2obj
for option, value in options:
    if option in ('--parameter1', '--p1'):
        parameter1 = eval(value)     # if not string
    elif option in ('--parameter2', '--p2'):
        parameter2 = value           # if string
    elif option in ('--parameter3', '--p3'):
        parameter3 = str2obj(value)  # if any object
```


Summary of eval and exec

- Evaluating string expressions with `eval`:

```
>>> x = 20
>>> r = eval('x + 1.1')
>>> r
21.1
>>> type(r)
<type 'float'>
```

- Executing strings with Python code, using `exec`:

```
exec("""
def f(x):
    return %s
""" % sys.argv[1])
```

Summary of exceptions

- Handling exceptions:

```
try:
    <statements>
except ExceptionType1:
    <provide a remedy for ExceptionType1 errors>
except ExceptionType2, ExceptionType3, ExceptionType4:
    <provide a remedy for three other types of errors>
except:
    <provide a remedy for any other errors>
...
```

- Raising exceptions:

```
if z < 0:
    raise ValueError\
          ('z=%s is negative - cannot do log(z)' % z)
```


A Summarizing example: solving $f(x) = 0$

- Nonlinear algebraic equations like

$$x = 1 + \sin x$$

$$\tan x + \cos x = \sin 8x$$

$$x^5 - 3x^3 = 10$$

  are usually impossible to solve by pen and paper
- Numerical methods can solve these easily
- There are general algorithms for solving $f(x) = 0$ for "any" f
- The three equations above correspond to

$$f(x) = x - 1 - \sin x$$

$$f(x) = \tan x + \cos x - \sin 8x$$

$$f(x) = x^5 - 3x^3 - 10$$

We shall learn about a method for solving $f(x) = 0$

- A solution x of $f(x) = 0$ is called a *root* of $f(x)$

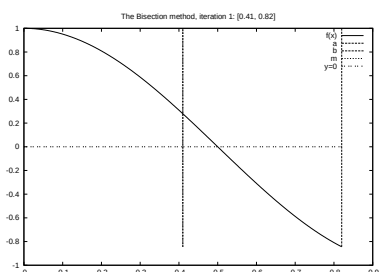
Outline of the the next slides:

- Formulate a method for finding a root
- Translate the method to a precise algorithm
- Implement the algorithm in Python

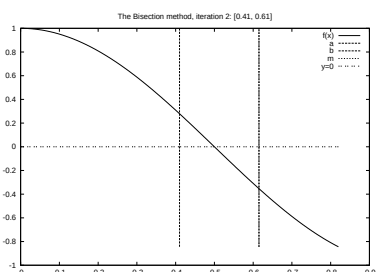
The Bisection method

- Start with an interval $[a, b]$ in which $f(x)$ changes sign
- Then there must be (at least) one root in $[a, b]$
- Halve the interval:
  - $m = (a + b)/2$; does f change sign in left half $[a, m]$?
  - Yes: continue with left interval $[a, m]$ (set $b = m$)
  - No: continue with right interval $[m, b]$ (set $a = m$)
- Repeat the procedure
- After halving the initial interval $[p, q]$ n times, we know that $f(x)$ must have a root inside a (small) interval $2^{-n}(q - p)$
- The method is slow, but very safe
- Other methods (like Newton's method) can be faster, but may also fail to locate a root – bisection does not fail

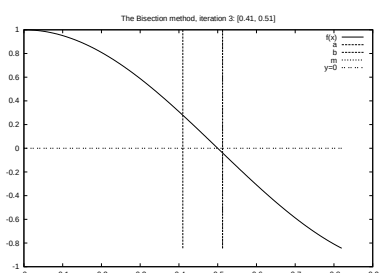
Solving $\cos \pi x = 0$: iteration no. 1



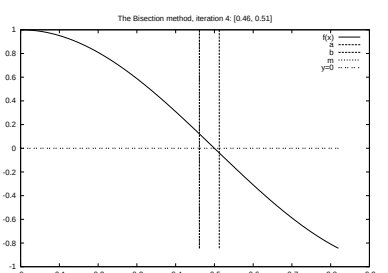
Solving $\cos \pi x = 0$: iteration no. 2



Solving $\cos \pi x = 0$: iteration no. 3



Solving $\cos \pi x = 0$: iteration no. 4



From method description to a precise algorithm

- We need to translate the mathematical description of the Bisection method to a Python program
- An important intermediate step is to formulate a precise algorithm
- Algorithm = detailed, code-like formulation of the method

for $i = 0, 1, 2, \ldots, n$
  $m = (a + b)/2$ (compute midpoint)
  if $f(a)f(m) \leq 0$ then
    $b = m$ (root is in left half)
  else
    $a = m$ (root is in right half)
  end if
end for
$f(x)$ has a root in $[a, b]$

The algorithm can be made more efficient

- $f(a)$ is recomputed in each if test
- This is not necessary if a has not changed since last pass in the loop
- On modern computers and simple formulas for $f(x)$ these extra computations do not matter
- However, in science and engineering one meets f functions that take hours or days to evaluate at a point, and saving some $f(a)$ evaluations matters!
- Rule of thumb: remove redundant computations (unless the code becomes much more complicated, and harder to verify)

New, more efficient version of the algorithm

Idea: save $f(x)$ evaluations in variables
$f_a = f(a)$
for $i = 0, 1, 2, \ldots, n$
  $m = (a + b)/2$
  $f_m = f(m)$
  if $f_a f_m \leq 0$ then
    $b = m$ (root is in left half)
  else
    $a = m$ (root is in right half)
    $f_a = f_m$
  end if
end for
$f(x)$ has a root in $[a, b]$

How to choose n? That is, when to stop the iteration

- We want the error in the root to be ϵ or smaller
- After n iterations, the initial interval $[a, b]$ is halved n times and the current interval has length $2^{-n}(b - a)$. This is sufficiently small if

$$2^{-n}(b-a) = \epsilon \quad \Rightarrow \quad n = -\frac{\ln \epsilon - \ln(b-a)}{\ln 2}$$

- A simpler alternative: just repeat halving until the length of the current interval is $\leq \epsilon$
- This is easiest done with a while loop: `while b-a <= epsilon`:
- We also add a test to check if f really changes sign in the initial inverval $[a, b]$

Final version of the Bisection algorithm

$f_a = f(a)$
if $f_a f(b) > 0$ then
  error: f does not change sign in $[a, b]$
end if
$i = 0$
while $b - a > \epsilon$:
  $i \leftarrow i + 1$
  $m = (a + b)/2$
  $f_m = f(m)$
  if $f_a f_m \leq 0$ then
    $b = m$ (root is in left half)
  else
    $a = m$ (root is in right half)
    $f_a = f_m$
  end if
end while
if x is the real root, $|x - m| < \epsilon$

Python implementation of the Bisection algorithm

```
def f(x):
    return 2*x - 3   # one root x=1.5

eps = 1E-5
a, b = 0, 10

fa = f(a)
if fa*f(b) > 0:
    print 'f(x) does not change sign in [%g,%g].' % (a, b)
    sys.exit(1)

i = 0   # iteration counter
while b-a > eps:
    i += 1
    m = (a + b)/2.0
    fm = f(m)
    if fa*fm <= 0:
        b = m  # root is in left half of [a,b]
    else:
        a = m  # root is in right half of [a,b]
        fa = fm
x = m          # this is the approximate root
```


Implementation as a function (more reusable $\Rightarrow$ better!)

```
def bisection(f, a, b, eps):
    fa = f(a)
    if fa*f(b) > 0:
        return None, 0

    i = 0   # iteration counter
    while b-a < eps:
        i += 1
        m = (a + b)/2.0
        fm = f(m)
        if fa*fm <= 0:
            b = m  # root is in left half of [a,b]
        else:
            a = m  # root is in right half of [a,b]
            fa = fm
    return m, i
```


Make a module of this function

- If we put the `bisection` function in a file `bisection.py`, we automatically have a module, and the `bisection` function can easily be imported in other programs to solve $f(x) = 0$
- Verification part in the module is put in a "private" function and called from the module's test block:

```
def _test():            # start with _ to make "private"
    def f(x):
        return 2*x - 3   # one root x=1.5

    eps = 1E-5
    a, b = 0, 10
    x, iter = bisection(f, a, b, eps)
    # check that x is 1.5

if __name__ == '__main__':
    _test()
```

To the point of this lecture: get input!

- We want to provide an $f(x)$ formula at the command line along with a and b (3 command-line args)
- Usage:

```
python bisection_solver.py 'sin(pi*x**3)-x**2' -1 3.5
```

The complete application program:

```
import sys
f_formula = sys.argv[1]
a = float(sys.argv[2])
b = float(sys.argv[3])
epsilon = 1E-6

from scitools.StringFunction import StringFunction
f = StringFunction(f_formula)

from bisection import bisection

root, iter = bisection(f, a, b, epsilon)
print 'Found root %g in %d iterations' % (root, iter)
```


Improvements: error handling

```
import sys
try:
    f_formula = sys.argv[1]
    a = float(sys.argv[2])
    b = float(sys.argv[3])
except IndexError:
    print '%s f-formula a b [epsilon]' % sys.argv[0]
    sys.exit(1)

try:  # is epsilon given on the command-line?
    epsilon = float(sys.argv[4])
except IndexError:
    epsilon = 1E-6  # default value

from scitools.StringFunction import StringFunction
from math import *  # might be needed for f_formula
f = StringFunction(f_formula)
from bisection import bisection
root, iter = bisection(f, a, b, epsilon)
if root == None:
    print 'No root found'; sys.exit(1)
print 'Found root %g in %d iterations' % (root, iter)
```


Applications of the Bisection method

- Two examples: $\tanh x = x$ and $\tanh x^5 = x^5$
- Can run a program for graphically demonstrating the method:

```
Unix/DOS> python bisection_plot.py "x-tanh(x)" -1 1
Unix/DOS> python bisection_plot.py "x**5-tanh(x**5)" -1 1
```

- The first equation is easy to treat
- The second leads to much less accurate results
- Why??? Run the demos!