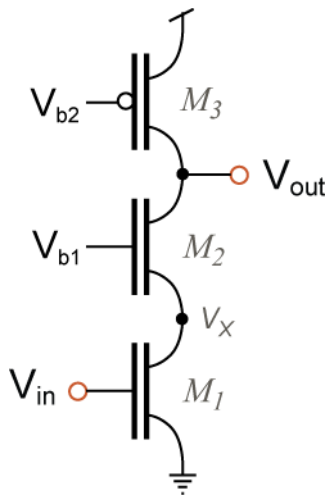


Solutions for inf3410 exam fall 2009



- 1) The amplifier of figure 1 is called a cascoded amplifier and is an inverting amplifier with high gain.
- 2) The M_1 transistor is the main common-source transistor amplifying the incoming signal by delivering a current on the output drain terminal. M_2 is a common-gate stage used in a cascode configuration in order to increase output impedance. The M_3 transistor is used as a current-source with a suitable biasing voltage for converting signal current to a suitable output voltage swing.

3) The circuit transfer function may be found by using the fact that the same current is flowing through all transistors:

$$g_{m1}V_{in} + \frac{V_x}{r_{ds1}} = \frac{V_{out} - V_x}{r_{ds2}} - g_{m2}V_x = -\frac{V_{out}}{r_{ds3}}$$

Using expression 2 and 3 we may find V_x :

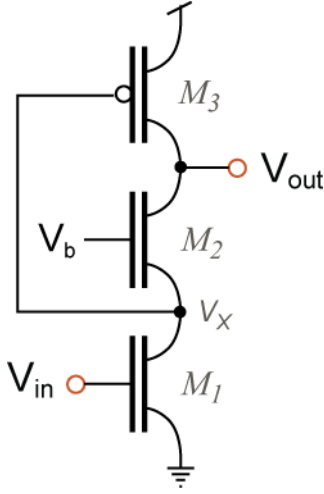
$$\begin{aligned} \frac{V_{out} - V_x}{r_{ds2}} - g_{m2}V_x &= -\frac{V_{out}}{r_{ds3}} \\ V_x \left(\frac{1}{r_{ds2}} + g_{m2} \right) &= V_{out} \left(\frac{1}{r_{ds3}} + \frac{1}{r_{ds2}} \right) \\ V_x &= V_{out} \frac{\left(\frac{1}{r_{ds3}} + \frac{1}{r_{ds2}} \right)}{\left(\frac{1}{r_{ds2}} + g_{m2} \right)} = V_{out} \frac{r_{ds2} + r_{ds3}}{r_{ds3}(g_{m2}r_{ds2} + 1)} \end{aligned}$$

Using current expression 1 and 3 and insert for V_x we get:

$$\begin{aligned} g_{m1}V_{in} + \frac{V_x}{r_{ds1}} &= -\frac{V_{out}}{r_{ds3}} \\ g_{m1}V_{in} + \frac{V_{out}}{r_{ds1}} \frac{r_{ds2} + r_{ds3}}{r_{ds3}(g_{m2}r_{ds2} + 1)} &= -\frac{V_{out}}{r_{ds3}} \\ g_{m1}V_{in} &= -V_{out} \left(\frac{1}{r_{ds1}} \frac{r_{ds2} + r_{ds3}}{r_{ds3}(g_{m2}r_{ds2} + 1)} + \frac{1}{r_{ds3}} \right) \\ \frac{V_{out}}{V_{in}} &= -\frac{g_{m1}r_{ds1}r_{ds3}(g_{m2}r_{ds2} + 1)}{r_{ds2} + r_{ds3} + r_{ds1}(g_{m2}r_{ds2} + 1)} \end{aligned}$$

The body-effect (back-gate) and front-gate are working "together" in the M2 transistor. For that reason we may substitute g_{m2} with $g_{m2} + g_{s2}$ in the transfer function above.

4) Transfer function:



Again we are equating currents through the transistors:

$$g_{m1}V_{in} + \frac{V_x}{r_{ds1}} = \frac{V_{out} - V_x}{r_{ds2}} - g_{m2}V_x = -\left(\frac{V_{out}}{r_{ds3}} + g_{m3}V_x\right)$$

Again we may combine the two last expression to find V_x :

$$\begin{aligned} \frac{V_{out} - V_x}{r_{ds2}} - g_{m2}V_x &= -\left(\frac{V_{out}}{r_{ds3}} + g_{m3}V_x\right) \\ V_x \left(\frac{1}{r_{ds2}} + g_{m2} - g_{m3}\right) &= V_{out} \left(\frac{1}{r_{ds3}} + \frac{1}{r_{ds2}}\right) \\ V_x &= V_{out} \frac{\frac{1}{r_{ds3}} + \frac{1}{r_{ds2}}}{\frac{1}{r_{ds2}} + g_{m2} - g_{m3}} \end{aligned}$$

Now substituting for V_x in the 1 and 3 expression we get:

$$\begin{aligned} g_{m1}V_{in} + \frac{V_x}{r_{ds1}} &= -\frac{V_{out}}{r_{ds3}} - g_{m3}V_x \\ g_{m1}V_{in} &= -\frac{V_{out}}{r_{ds3}} - \left(g_{m3} + \frac{1}{r_{ds1}}\right)V_{out} \frac{\frac{1}{r_{ds3}} + \frac{1}{r_{ds2}}}{\frac{1}{r_{ds2}} + g_{m2} - g_{m3}} \\ g_{m1}V_{in} &= -V_{out} \left(\frac{(r_{ds2} + r_{ds3})(g_{m3}r_{ds1} + 1)}{r_{ds1}r_{ds3}((g_{m2} - g_{m3})r_{ds2} + 1)} + \frac{1}{r_{ds3}}\right) \end{aligned}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{ds1}r_{ds3}((g_{m2} - g_{m3})r_{ds2} + 1)}{(r_{ds2} + r_{ds3})(g_{m3}r_{ds1} + 1) + r_{ds1}((g_{m2} - g_{m3})r_{ds2} + 1)}$$

The body-effect (back-gate) and front-gate are working "together" in the M2 transistor. For that reason we may substitute g_{m2} with $g_{m2} + g_{s2}$ in the transfer function above.

- 5) In order to compare voltage gain from the circuits above we may assume all devices have the same transconductance and drain resistance. Simplifying transfer function in question 3 and inserting $g_{m2} + g_{s2}$ we get:

$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_{ds}(g_m r_{ds} + g_{s2} r_{ds} + 1)}{g_m r_{ds} + g_{s2} r_{ds} + 3} \approx -g_m r_{ds}$$

And from question 4 we get:

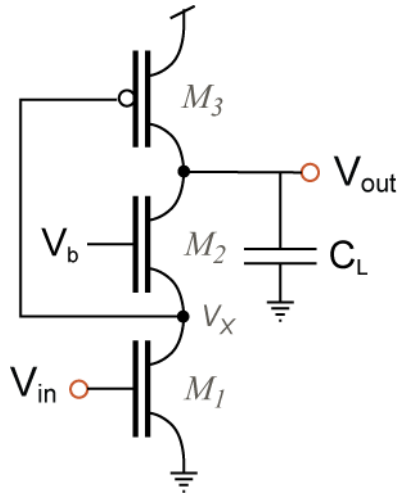
$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_{ds}(g_{s2} r_{ds} + 1)}{2g_m r_{ds} + g_{s2} r_{ds} + 3} \approx -\frac{g_m g_{s2} r_{ds}}{2g_m + g_{s2}}$$

Assuming $g_m \approx 10g_{s2}$:

$$\frac{V_{out}}{V_{in}} \approx -\frac{g_m g_{s2} r_{ds}}{2g_m + g_{s2}} \approx -\frac{10g_{s2} g_{s2} r_{ds}}{20g_{s2} + g_{s2}} \approx -\frac{1}{2} g_{s2} r_{ds}$$

Comparing these expressions we find that the voltage gain found in question 3 is about 20 times larger than the voltage gain found in question 4.

6) In order to determine the frequency dependant transfer function of the circuit



We may use equations from problem 4 above and add the capacitive load on output:

$$g_{m1}V_{in} + \frac{V_x}{r_{ds1}} = \frac{V_{out} - V_x}{r_{ds2}} - g_{m2}V_x = -\left(\frac{V_{out}}{r_{ds3}} + g_{m3}V_x + V_{out}C_Ls\right)$$

Solving using the same procedure as above we get:

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{ds1}r_{ds3}((g_{m2}-g_{m3})r_{ds2} + 1)}{(r_{ds2} + r_{ds3})(g_{m3}r_{ds1} + 1) + r_{ds1}((g_{m2}-g_{m3})r_{ds2} + 1) + r_{ds1}r_{ds3}((g_{m2}-g_{m3})r_{ds2} + 1)C_Ls}$$

Inserting $g_{m2} + g_{s2}$ for g_{m2} , the body effect of M2 is included. Assuming transistor characteristics are equal we may simplify:

$$\begin{aligned} H(s) &= \frac{V_{out}}{V_{in}} \\ &= -\frac{g_m r_{ds} r_{ds} ((g_m + g_{s2} - g_m) r_{ds} + 1)}{(r_{ds} + r_{ds})(g_m r_{ds} + 1) + r_{ds} ((g_m + g_{s2} - g_m) r_{ds} + 1) + r_{ds} r_{ds} ((g_m + g_{s2} - g_m) r_{ds} + 1) C_L s} \\ &= -\frac{g_m r_{ds} (g_{s2} r_{ds} + 1)}{2g_m r_{ds} + 2 + g_{s2} r_{ds} + 1 + r_{ds} (g_{s2} r_{ds} + 1) C_L s} \end{aligned}$$

Again simplifying we get:

$$\begin{aligned} H(s) &= \frac{V_{out}}{V_{in}} \approx -\frac{g_m g_{s2} r_{ds}}{2g_m + g_{s2} + r_{ds} g_{s2} C_L s} \\ &= \frac{\frac{g_m g_{s2} r_{ds}}{2g_m + g_{s2}}}{1 + \frac{r_{ds} g_{s2} C_L}{2g_m + g_{s2}} s} = \frac{A_0}{1 + s/\omega_0} \end{aligned}$$

$$\text{With } A_0 = \frac{g_m g_{s2} r_{ds}}{2g_m + g_{s2}} \text{ and } \omega_0 = \frac{2g_m + g_{s2}}{r_{ds} g_{s2} C_L}$$