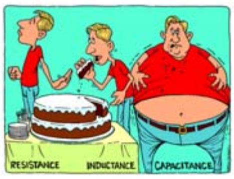
# Frequency response



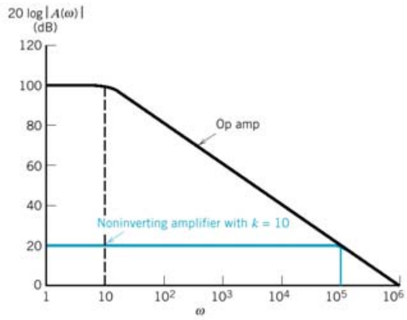

- Static analysis
  - Low frequency response
  - Response without capacitive effect
- Real circuit response
  - Strongly dependant on frequency
  - Small signal analysis with (major) capacitances (and inductors for RF)

Typical operational amplifier response

- Gain strongly dependant on frequency

Frequency response

- Important performance measure
  - Gain-Bandwidth

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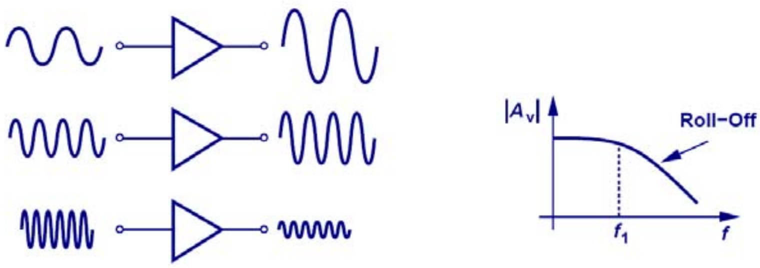
# Frequency response

- Gain changing with signal frequency




(a) (b)

  - Sinusoidals may be combined to construct any signal

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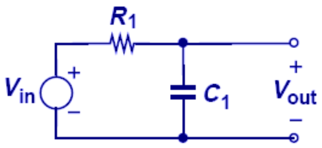
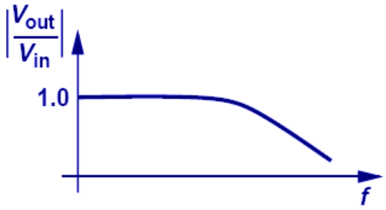
# Simple low-pass filter

- Passive filter



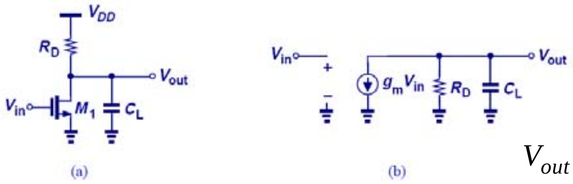




  - $C_1$
    - high impedance a low frequency
    - Reduced impedance with increasing frequyency



# CS stage with load

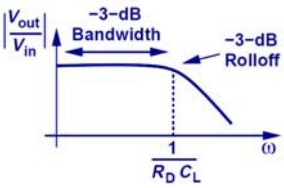
- Finding transfer function




$$V_{out} = -g_m V_{in} \left( R_D \,||\, \frac{1}{C_L s} \right)$$

  - Simply by adding capacitance impedance to output load

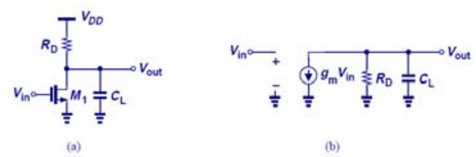
- magnitude




$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

# Bandwidth

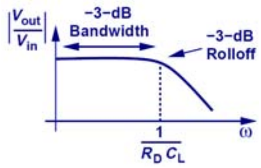
- Example




$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m(R_D \,||\, \frac{1}{C_L s})$$
$$= \frac{-g_m R_D}{R_D C_L s + 1}.$$

$$s = j\omega \rightarrow \quad \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}.$$

$$\omega = 1/(R_D C_L) \quad \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{2}}.$$



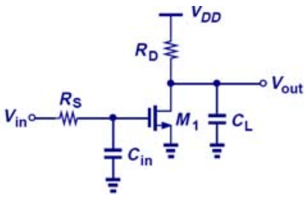

Time constant or -3dB bandwidth



# Nodes and poles

- Poles are often associated to nodes

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

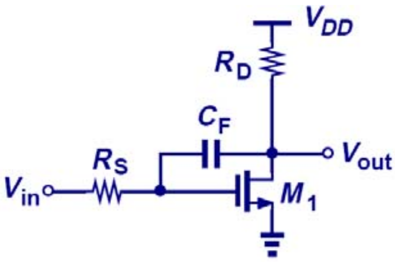



$$|\omega_{p1}| = \frac{1}{R_S C_{in}} \qquad |\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

# What about this?

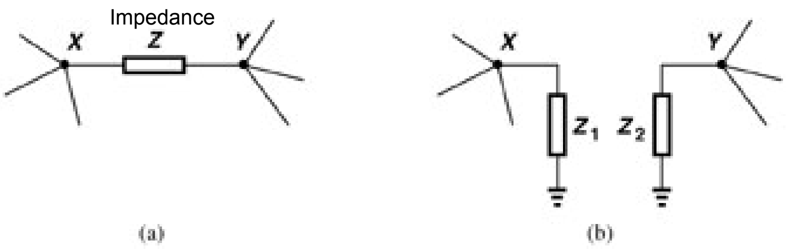
- CS stage with feedback




  - Nodes connected by floating capacitor
  - Simple node for each pole does not work
  - Must transform before analysis
    - Caps to ground…..



# Miller Theorem



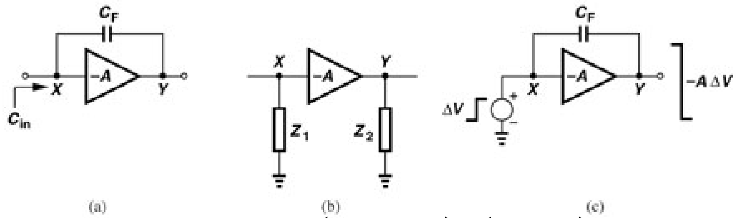

- If circuit (a) is converted to (b) then:

$$Z_1 = \frac{Z}{1 - A_v} \qquad Z_2 = \frac{Z}{\left(1 - \frac{1}{A_v}\right)} \qquad A_v = \frac{V_Y}{V_X}$$

*Approximation!*

# Example

- Ideal amp with feedback capacitor




$$Z = 1/sC_F \Rightarrow Z_1 = (1/sC_F)/(1+A)$$

$$\Rightarrow C_1 = C_F(1+A)$$

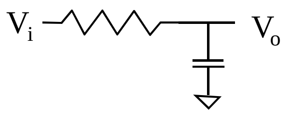
$$Z = 1/sC_F \Rightarrow Z_2 = (1/sC_F)/(1+1/A)$$

$$\Rightarrow C_2 = C_F(1+A^{-1}) \approx C_F$$



# Simple 1. order filter

- Passive




$$V_o/V_i = \frac{1/sC}{R+1/sC}$$

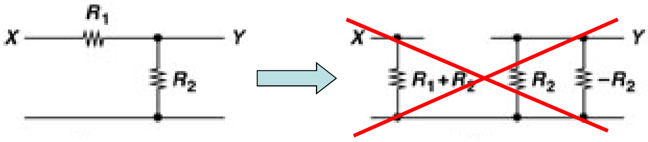
$$V_o/V_i = \frac{1}{sRC+1}$$

$$\frac{V_o}{V_i}(f) = \frac{1}{1+(j2\pi f)RC}$$

$$\frac{V_o}{V_i}(f) = \frac{1}{1+j\,f/f_p}\ , \quad f_p = 1/2\pi RC$$

# Limited usage

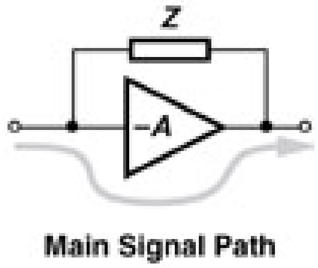
- Do not work for passive voltage divider




- input impedance correct
- Gain wrong

- Typical case for Miller theorem

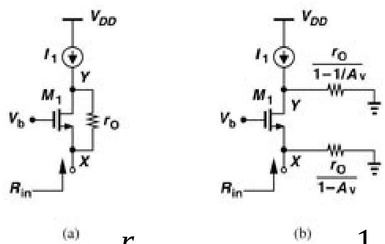
Feedback in parallel with main signal






# Example

- Determine input impedance using Miller




$$R_{in} = \frac{r_o}{1-[1+(g_m+g_{mb})r_o]} \,||\, \frac{1}{g_m+g_{mb}}$$

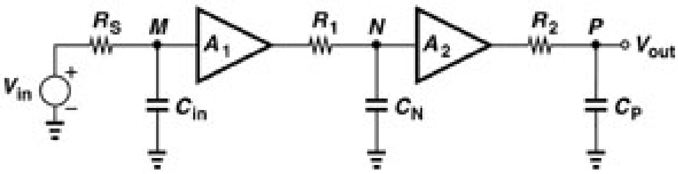
$$= \frac{-1}{g_m+g_{mb}} \,||\, \frac{1}{g_m+g_{mb}}$$

$$= \infty$$

Determined in chapter 3 as well

# Poles and nodes

- Assuming one pole for each node




– Total gain by multiplying each stage gain

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+sR_sC_{in}} \cdot \frac{A_2}{1+sR_1C_N} \cdot \frac{1}{1+sR_2C_P}$$

*Simple and easy 1. order approximation*

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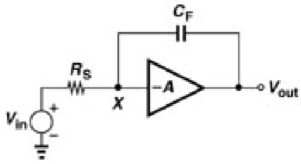
# Example

- Determine pole at node X




– Using Miller to find input impedance to ground:

$$Z_1 = (1+A)C_F$$

– Time-constant by multiplying by $R_s$

$$f_{p-in} = \frac{1}{2\pi(R_S \cdot (1+A)C_F)} \qquad / Hz$$

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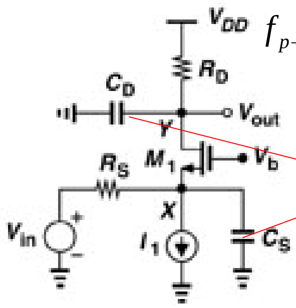
# Example

- More complicated to appreciate nodal analysis
  - Neglecting channel length modulation




$$f_{p-in(X)} = \frac{1}{2\pi\left[(C_{GS}+C_{SB})\left(R_S \| \left(\frac{1}{g_m+g_{mb}}\right)\right)\right]}$$

$$f_{p-out(Y)} = \frac{1}{2\pi[(C_{GD}+C_{DB})R_D]}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m+g_{mb})R_D}{1+(g_m+g_{mb})R_S} \cdot \frac{1}{(1+s/2\pi f_{p-in(X)})\cdot(1+s/2\pi f_{p-out(Y)})}$$

With channel length modulation, things are hard due to input-output interaction

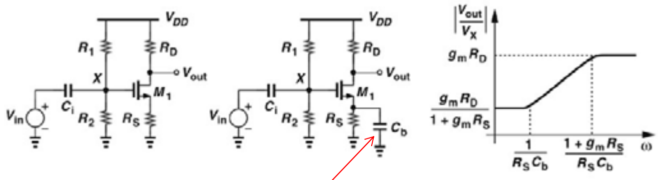


# Analysis procedure

1. Determine capacitors affecting low-frequency response
2. Calculate mid-band gain by shorting these caps
3. Identify and add transistors capacitors
4. Note AC ground and try to get rid of caps not affecting performance
5. Determine high frequency poles and zeros
6. Plot frequency response or find equation

# Low frequency response

- CS stage




  – Transfer function: (a) (b) (c)

$$V_{out}/V_{in} = (V_{out}/V_X)(V_X/V_{in})$$

$$V_{out}/V_X = -R_D/(R_S + 1/g_m)$$

$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \| R_2}{R_1 \| R_2 + \frac{1}{C_i s}} = \frac{(R_1 \| R_2)C_i s}{(R_1 \| R_2)C_i s + 1}$$

  – High pass filter on input
    - Cut-off below lowest signal frequency

$$\frac{1}{2\pi[(R_1 \| R_2)C_i]} < f_{sig,min}$$

  – Bypass capacitor
    - Removing degeneration for mid-band frequencies

$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S \| \frac{1}{C_b s} + \frac{1}{g_m}} = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}.$$

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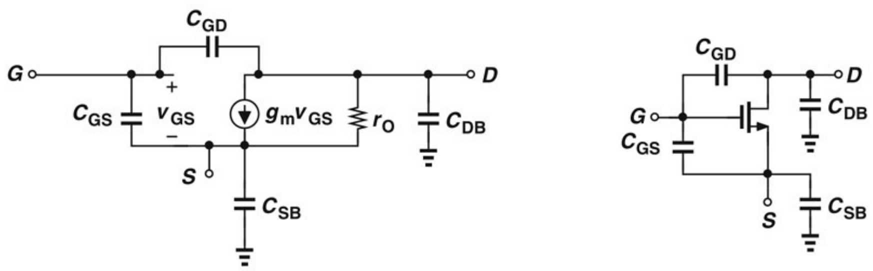
# High frequnecy MOS model

- Adding transistor capacitances




(a) (b)

  – $C_{GD}$ appear as a floating or coupling capacitance
  – Other caps decoupling
    - Except when source is not ground connected

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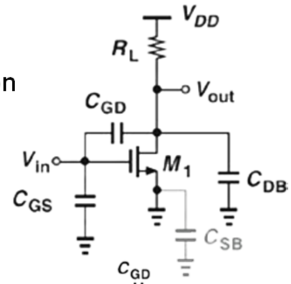
# High frequency response

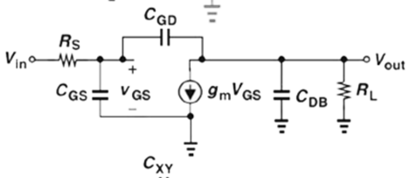



- Using Millers theorem
  - Ignoring channel length modulation

$$C_X = (1+g_m R_L)C_{XY}$$

$$C_Y = \left(1+\frac{1}{g_m R_L}\right)C_{XY}$$

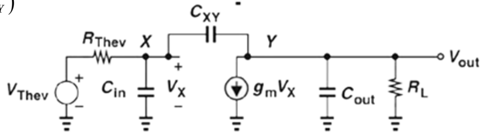
- Poles at each node




$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in}+(1+g_m R_L)C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L\left[C_{out}+\left(1+\frac{1}{g_m R_L}\right)C_{XY}\right]} \approx \frac{1}{R_L(C_{out}+C_{XY})}$$

$g_m R_L \gg 1$

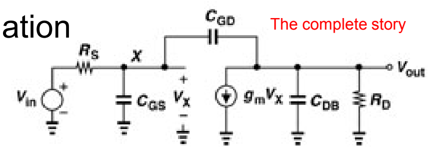



$$\frac{V_{out}}{V_{in}}(s) = \overbrace{-g_m R_D}^{\text{LF gain}} \frac{1}{(1+s/\omega_{p,in})\cdot(1+s/\omega_{p,out})}$$



# CS small signal analysis

- Dominant pole approximation
  - Using KCL




- Node X: $$\frac{V_X - V_{in}}{R_s} + V_X C_{GS} s + (V_X - V_{out})C_{GD} s = 0$$
- Node $V_{out}$: $$(V_{out} - V_X)C_{GD} s + g_m V_X + V_{out}(1/R_D + C_{DB} s) = 0 \Rightarrow$$
- Insert: $$V_X = -\frac{V_{out}\left(sC_{GD} + \frac{1}{R_D} + sC_{DB}\right)}{g_m - sC_{GD}}$$

$$-V_{out}\frac{[R_S^{-1}+(C_{GS}+C_{GD})s][R_D^{-1}+(C_{GD}+C_{DB})s]}{g_m - C_{GD}s} - V_{out}C_{GD}s = \frac{V_{in}}{R_s}$$

# CS small signal analysis

- Giving transfer function:

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{s^2(R_S R_D \xi) + s[R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})] + 1}$$

  - With $\xi = C_{GD}.C_{GS} + C_{GS}.C_{DB} + C_{GD}.C_{GB}$
- Assuming two poles (dominant pole approximation)

$$D = \left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1 \qquad (\omega_{p1} << \omega_{p2})$$

$$\omega_{p1} = \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}$$

$$\omega_{p,in} = \frac{1}{R_S[C_{GS} + (1+g_m R_D)C_{GD}]}$$

Addition compared to simple Miller analysis.
This is correct with good pole separation




# CS small signal analysis

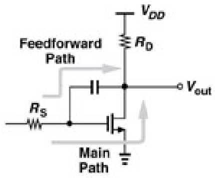
- Determining second pole
  - Given s² coefficient is $\frac{1}{(\omega_{p1}\omega_{p2})}$ ⇨ $\omega_{p2} = \frac{1}{\omega_{p1}}\frac{1}{R_S R_D \xi}$

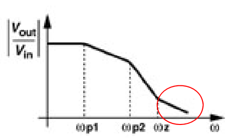
$$\omega_{p2} = \frac{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}{R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

Miller approximation

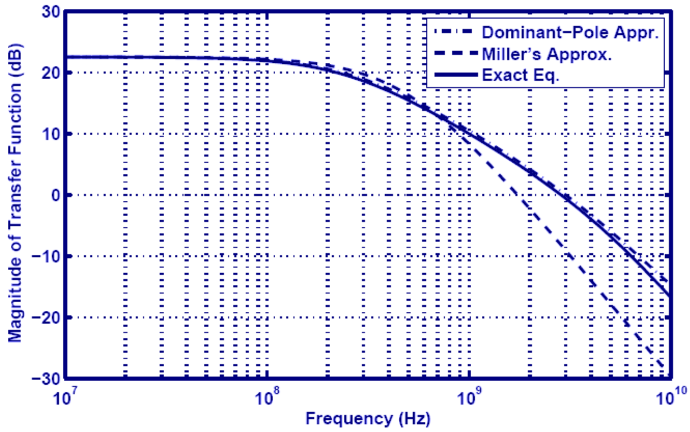
Assuming: $C_{GS} \gg (1+g_m R_D)C_{GD} + {R_D(C_{GD}+C_{DB})}/{R_S}$ ⇨ $\omega_{p2} = \omega_{p,out} \approx \frac{1}{(C_{GD}+C_{DB})R_D}$

- Zero at $\omega_z = +g_m / C_{GD}$
  - Feed forward at high frequencies
    - Not so steep transition band
  - Not predicted by Miller
  - May introduce instability
    - With feedback



# Comparison of approximations

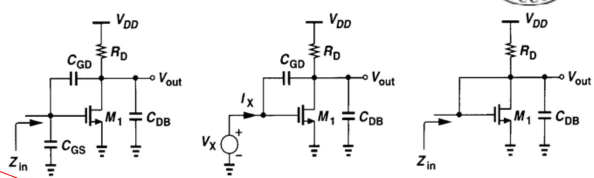



- Dominant pole approximation is reasonably good
- Miller approximation have significant errors at higher frequencies



# CS input impedance

- HF input impedance




  - a)

$$Z_{in} = \frac{1}{[C_{GS} + (1+g_m R_D)C_{GD}]s}$$

  - b) compensating for output node variations ($C_{GS}$ ignored)

$$V_X = (I_X - g_m V_X)\frac{R_D}{1+R_D C_{GB} s} + \frac{I_X}{C_{GD} s}$$

Voltage change over $C_{GD}$

Voltage change at output perturbations

$$\frac{V_X}{I_X} = \frac{1+R_D(C_{GD}+C_{GB})s}{C_{GD}s(1+g_m R_D + R_D C_{GB} s)}$$

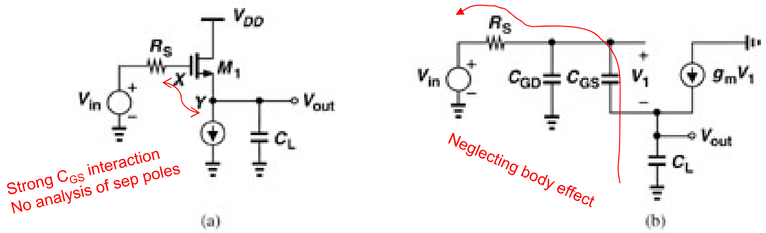
Total input impedance $C_{GS}$ must be added in parallel

When $R_D(C_{GD}+C_{GB})s << 1$ and $R_D C_{GB} s << 1 + g_m R_D$ reduce to simple form

Load mostly capacitive!

# Source followers freq

- Level shifters and buffers




- Summing currents

$$V_1 C_{GS} s + g_m V_1 = V_{out} C_L s \Rightarrow \quad V_1 = \frac{C_L s}{g_m + C_{GS} s} V_{out} \qquad \text{KCL}$$

$$V_{in} = R_S[\underbrace{V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s}_{I_{Rs}}] + V_1 + V_{out} \qquad \text{KVL}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s(g_m R_S C_{GD} + C_L + C_{GS}) + g_m} \qquad \text{combining}$$

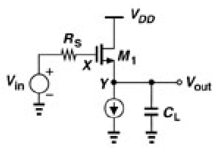


# Follower instability

- Zero in left half-plane

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s(g_m R_S C_{GD} + C_L + C_{GS}) + g_m}$$

- Assuming separated poles
  - Dominating pole




$$f_{p1} \approx \frac{g_m}{2\pi(g_m R_S C_{GD} + C_L + C_{GS})}, \text{ assuming } f_{p2} >> f_{p1}$$

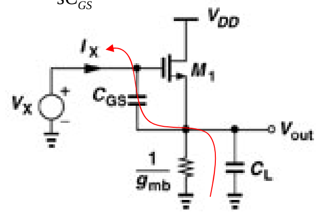
$$= \frac{1}{2\pi\left(R_S C_{GD} + \dfrac{C_L + C_{GS}}{g_m}\right)} \qquad R_S = 0 \rightarrow f_{p1} \approx \frac{g_m}{2\pi(C_L + C_{GS})}$$

# Source follower input impedance

- Ignoring $C_{GD}$

$$V_X = \frac{I_X}{sC_{GS}} + \overbrace{\left(I_X + \frac{g_m I_X}{sC_{GS}}\right)}^{I_{total}}\left(\frac{1}{g_{mb}} \| \frac{1}{sC_L}\right)$$

$$V_{GS} = I_X \frac{1}{sC_{GS}}$$




- Assuming $g_{mb} >> |sC_L|$

$$Z_{in} = \frac{1}{sC_{GS}} + \left(1 + \frac{g_m}{sC_{GS}}\right)\frac{1}{g_{mb} + sC_L}$$

$$Z_{in} \approx \frac{1}{sC_{GS}}(1 + g_m / g_{mb}) + 1/g_{mb}$$

  - Indicating input capacitance: $C_{in} = C_{GS}[g_{mb}/(g_m + g_{mb})] + C_{GD}$
  - At HF: $g_{mb} << |sC_L|$

Negative resistance for oscillators

$$Z_{in} = \frac{1}{sC_{GS}} + \left(1 + \frac{g_m}{sC_{GS}}\right)\frac{1}{\cancel{g_{mb}} + sC_L} \rightarrow \quad Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2 C_{GS} C_L}$$

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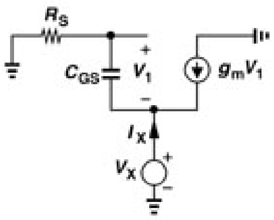
# Source follower output impedance

- Ignoring body effect and $C_{GD}$

$$V_1 C_{GS} s + g_m V_1 = -I_X$$

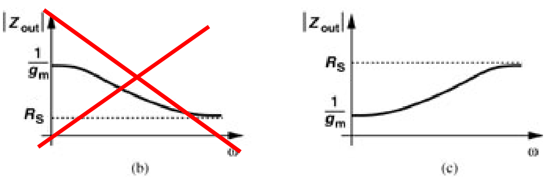
$$(V_1 C_{GS} s) R_S + V_1 + V_X = 0$$




  - Giving $Z_{OUT} = V_X / I_X$

$$Z_{OUT} = \frac{sR_S C_{GS} + 1}{g_m + sC_{GS}}$$

    - Magnitude function of frequency!
  - With $1/g_m < R_S$ for low output impedance (buffer)
    - Increasing impedance with frequency → inductive component!

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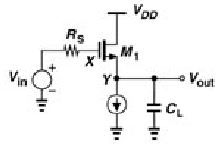
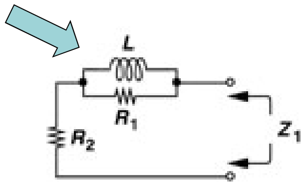
# SF Inductive component

- Assuming inductor
  - Replace with passive inductor equivalent




  - Giving $R_2 = 1/g_m$

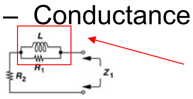
$$R_1 = R_S - 1/g_m$$

  - Finding L

$Z_{out}$

$$L \,||\, R_1 \Rightarrow Z_{out} - 1/g_m = \frac{sR_SC_{GS}+1}{g_m + sC_{GS}} - 1/g_m = \frac{sC_{GS}(R_S - 1/g_m)}{g_m + sC_{GS}}$$

  - Conductance



$$\frac{1}{Z_{out} - 1/g_m} = \frac{1}{Rs - 1/g_m} + \frac{1}{s\frac{C_{GS}}{g_m}(Rs - 1/g_m)}$$

$R_1$ $Z_L$

$$\Rightarrow L = \frac{C_{GS}}{g_m}(R_S - 1/g_m)$$

Inductive property dependant on $R_S$

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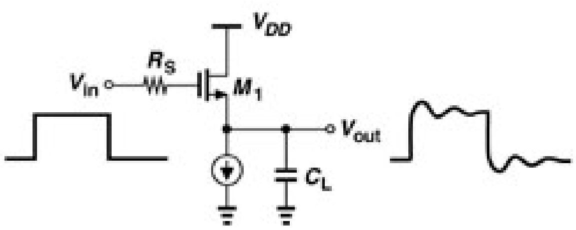
TSL inf3410

# Source follower instability

- For large $R_S$




  - Ringing on signal
    - Should be avoided

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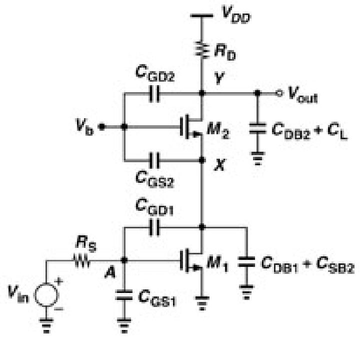
# Cascode stage frequency response

- Cascode
  - Cascaded CS+CG
  - High gain of CS
    - High impedance input
  - High speed of CG
    - Suppressing Miller effect
  - Node capacitances
    - Node A: $C_{GS1}$ to gnd and $C_{GD1}$
    - Node X: $C_{DB1}+C_{SB2}+C_{GS2}$
    - Node Y: $C_{GD2}+C_{DB2}+C_{L}$
  - Gain A→X: $A=G_m R_{out}=-g_{m1}\dfrac{1}{g_{m2}+g_{mb2}}\approx 1$
    - Assuming similar size
    - Mille cap: $(1+A)C_{GD1}\approx 2C_{GD1}$ *Ignorable Miller capacitance*






# Cascode poles

- Pole in node A

$$\omega_{p,A}=\frac{1}{R_S\left[C_{GS1}+\left(1+\dfrac{g_{m1}}{g_{m2}+g_{mb2}}\right)C_{GD1}\right]}$$

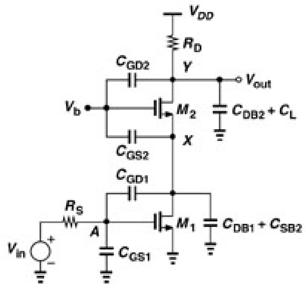
- Pole in node X

$$\omega_{p,X}=\frac{g_{m2}+g_{mb2}}{2C_{GD1}+C_{DB1}+C_{SB2}+C_{GS2}}$$

- Pole in output node

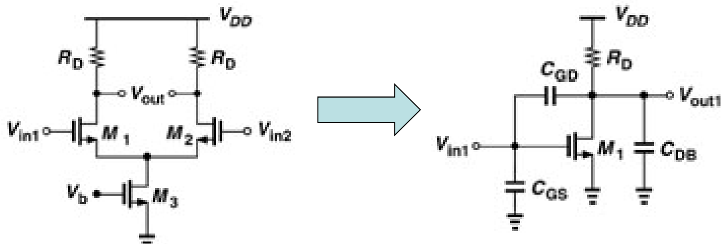
$$\omega_{p,Y}=\frac{1}{R_D\left(C_{DB2}+C_L+C_{GD2}\right)}$$

  - Pole in X are often moved upwards in frequency for stability



# Differential pair freq. response

- Analyze by half-circuit

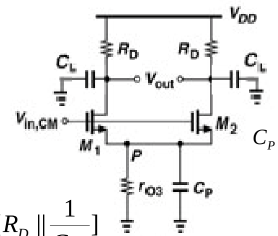



  - Response identical to CS response
  - Number of poles is symmetrical
    - Analyze poles of half-cricuit



# Differential pair CM freq. response

- Common-mode
  - $M_1$ – $M_3$ large devices
  - Substantial load at P
- Common-mode to differential error




$$C_P = C_{GD3} + C_{DB3} + C_{SB1} + C_{SB2}$$

$$A_{V,CM-DM}(s) = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{(g_{m1} - g_{m2})[R_D \,\|\, \frac{1}{C_L s}]}{(g_{m1} + g_{m2})[r_{o3} \,\|\, \frac{1}{C_P s}] + 1} =$$

$$A_{CM-DM} = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{(g_{m1} - g_{m2})R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

$$R_D \to R_D \,\|\, \frac{1}{C_L s} \quad R_{SS} \to r_{o3} \,\|\, \frac{1}{C_P s}$$

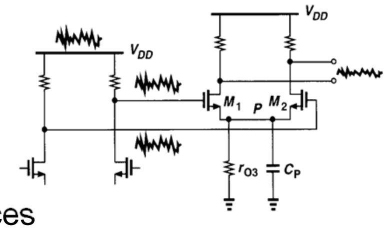
$$= -\frac{(g_{m1} - g_{m2})R_D}{(g_{m1} + g_{m2})r_{o3} + 1} \cdot \frac{(1 + s r_{o3} C_P)}{(1 + s R_D C_L)(1 + s \frac{r_{o3} C_P}{(g_{m1} + g_{m2})r_{o3} + 1})}$$

  - Due to input device mismatch
  - Output pole >> pole at P → poor CMRR
  - Zero dominates

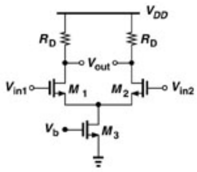
$$A_{CM-DM} \text{ raising } \omega > \frac{1}{r_{o3} C_P}$$

## Diff-pair poles

- P poles << output poles
  - Poor CMRR



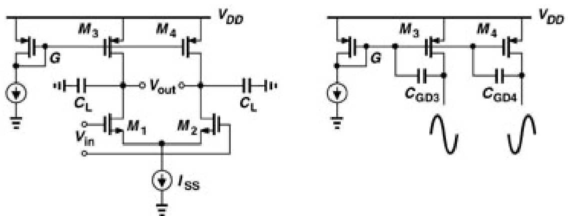

  - Often due to large devices
    - Maintain headroom






## Diff-pair with high-impedance load

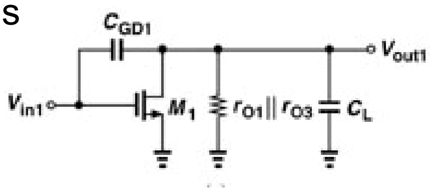
- Analyzing differential and CM separately
  - $C_L$ includes $C_{GD}$ and $C_{DB}$ of PMOS loads




  - For differential outputs, $C_{GD3}$ and $C_{GD4}$ conduct equal and opposite currents to node G
  - Therefore node G is ground for small-signal analysis
  - In practice: We hook up by-pass capacitor between G and ground

# Half-circuit analysis

- Using previous analysis




$R_D \rightarrow r_{o1} \| r_{o3}$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS}C_{GD} + C_{GS}C_{SB} + C_{GD}C_L) + s[R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_L)] + 1}$$

Eq 6.23

- Dominant pole

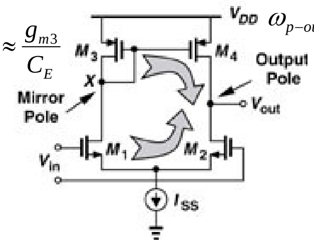
$$\frac{1}{(r_{o1} \| r_{o3})C_L}$$

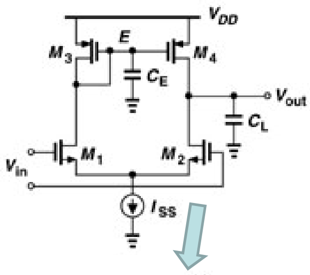


# Diff pair with active load

- Poles in two signal paths

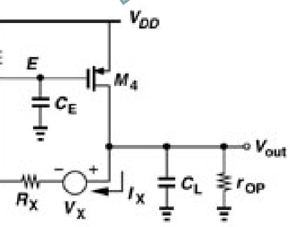
$$\omega_{p-E} \approx \frac{g_{m3}}{C_E} \qquad \omega_{p-out} \approx \frac{1}{(r_{oN} \| r_{oP})C_L}$$




- Frequency response

$$V_E = (V_{out} - V_X)\frac{\dfrac{1}{sC_E + g_{mP}}}{\dfrac{1}{sC_E + g_{mP}} + R_X}$$

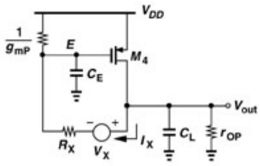



$R_X = 2r_{ON}$

$V_X = g_{mN} r_{ON} V_{in}$

Assuming: $g_{mP} << r_{OP}$

- Using KCL




$$V_{out}(sC_L + r_{oP}^{-1}) + g_{m4}V_E + I_X = 0$$

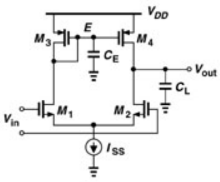
$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}r_{ON}(2g_{mP} + sC_E)}{s^2(2r_{OP}r_{ON}C_EC_L) + s[(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP}r_{ON})C_L] + 2g_{mP}(r_{ON} + r_{OP})}$$

- Mirror pole often higher in magnitude

$$\omega_{p-out} \approx \frac{2g_{mP}(r_{ON} + r_{OP})}{[(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP}r_{ON})C_L]} \approx \frac{1}{(r_{ON} \| r_{OP})C_L}$$

  - Neglecting first term in denominator
  - And $2g_{mP}r_{ON} >> 1$

$$\omega_{p-E} \approx \frac{g_{mP}}{C_E}$$




- Zero
  - Due to slow+fast signal path

$$\omega_Z = \frac{2g_{mP}}{C_E}$$



- Classic transfer function (2. order)
  - Determine stability
  - $\omega_0$ -- pole frequency
  - Q – Quality factor

$$A(s) = A(0)\frac{N(s)}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0}}$$

$$Q < \sqrt{1/2} = 0.707$$

- Unconditionally stable
- Max magnitude at DC

$$Q = \sqrt{1/2}$$

- -3db frequency is $\omega_0$

$$Q > 0.5$$

- Ringing overshoot

$$\%overshoot = 100e^{-\pi\sqrt{4Q^2-1}}$$

- No overshoot (only real poles)

$$Q < 0.5$$