Simulation of nonlinear fields from 2D ultrasound transducers and a new secondary grating lobe phenomenon

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Abstract—**In this study we have used Burgers' equation to study the field from sparse ultrasound arrays where the full array had 50 by 50 elements. The two-way sidelobe level was reduced by 20-25 dB compard to the fundamental mode. We have also found that a new grating lobe appears at the second harmonics at half the sine of the angle of the fundamental grating lobe.**

I. Introduction

In this study we wanted to simulate the two-way response from sparse arrays when imaging in a nonliner medium. The purpose was to compare the performance under nonlinarity with that in a linear medium as found in [1].

II. Theory

We have used Burgers' equation [2] to describe the acoustic field:

$$\frac{\partial u}{\partial z} = \frac{\beta \omega_o}{c_o^2} u \frac{\partial u}{\partial \tau} + \Gamma \frac{\partial^2 u}{\partial \tau^2}. \quad (1)$$

In this expression, c_o is the nominal speed of sound in the medium (speed of infinitesimal bulk waves), $\beta = 1 + \frac{B}{2A}$ where $\frac{B}{A}$ is the ratio of the first two terms in the nonlinear pressure - density relation for the medium, and, finally, Γ is a constant related to the thermo-viscous dissipation of the medium. Also, $\tau = \omega t - kz$.

The expression can be modified for numerical implementation [3]:

$$\frac{\partial u_n}{\partial z} = i \frac{\beta \omega_o}{4 c_o^2} \Big(\sum_{m=1}^{n-1} m U_m U_{n-m} + \sum_{m=n}^{\infty} n U_m U_{n-m}^* \Big). \quad (2)$$

The infinite series has to be truncated to a finite number of harmonics, N. The simulator is based on the principles of [4] and with modifications from [3] in order to generalize it from a circular source geometry to a rectangular geometry. In the simulator, nonlinear propagation is taken care of through the truncated eq. 2 and diffraction is handled from one depth to another through the angular spectrum method [5]. Attenuation can also be introduced in the diffraction step, although not used in our study.

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Transducer:	
Beam type	CW
Radius	7.5 mm
Initial pressure	600 kPa
Frequency	3 MHz
Elevation focus	80 mm
Azimuth focus	80 mm
Wavelength	0.5133 mm
Medium:	
Velocity of sound	1540 m/s
Non-lin. parameter (beta)	3.5
Attenuation	0 dB/cm/MHz
Number of harmonics	4
Number of lateral samples	640
Density	1000 kg/m^3
Maximum depth	100 mm
Sampling in z	0.39 mm
Sampling in x and y	0.308 mm

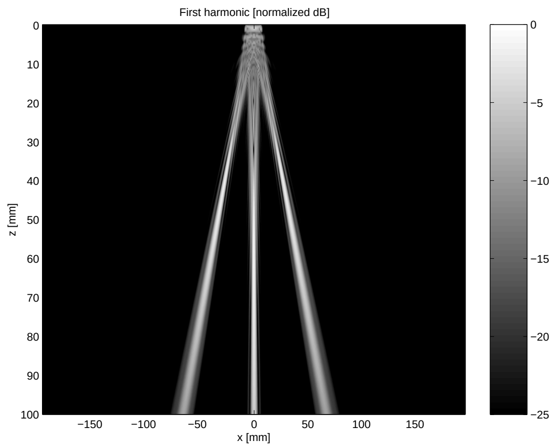
TABLE I
Simulation parameters.

III. Results

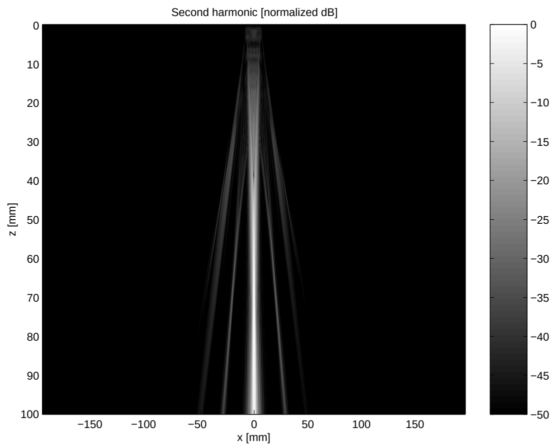
The main simulations are done for a set of sparse array layouts that were developed during the project "Real-time 3D Ultrasound Imaging System with Advanced Transducer Arrays (NICE)" where the objective was to lay the foundation of a real-time 3D imaging ultrasound system. Part of this project has been done at this University, specifically the construction of the sparse transducer for fundamental imaging. In the NICE project the geometries were evaluated by simulation and experiment for linear wave propagation [1].

When using harmonic imaging, the transmitted frequency has to be lowered because the double frequency has to be within the transducer bandwidth. This has not been done in these simulations because the grating lobes are moved to other locations and it is not possible to compare with conventional imaging.

The specifications used here are the same as those used in the project. The center frequency is 3 MHz and 308 μm pitch (0.6 times the nominal wavelength) and the 2D array has 50x50 elements. No apodization is used, and the corner elements are removed to approximate a circular array. The parameters used in this simulation are summarized in table I. Only the geometry is changed in the different simulations.



(a) First harmonic.



(b) Second harmonic.

Fig. 1. Nonlinear grating lobes

IV. Response from a point scatterer

A first order approximation to calculate the pulse-echo response is used. It is based on the product of the continuous wave of the transmit and the receive responses, [6].

The two-way response was calculated by assuming nonlinear propagation towards a point scatterer, and linear propagation back to the transducer. The linear propagation of the reflected beam is a valid assumption. The reason for this is that the reflected signal is so weak that it generates almost no distortion of this signal. Simulation of the receiving frequency is set to the double of the transmission frequency because it is the second harmonic that is interesting to study. This way to calculate the pulse-echo is valid in the focus of the beam.

The receive and transmit fields are multiplied and compared to a two-way fundamental image.

V. Nonlinear grating lobes

When using the nonlinear simulator, a new grating lobe was found at the second harmonic. This new lobe is a kind of grating lobe and appears at half of the sine to the angle of the fundamental.

It is known that fingers are created at the second harmonic. The first report on fingers is from 1973. They were found experimentally by Lockwood et al. [7], and later explained theoretically by Tjøtta et al. [8]. Fingers appear at the second harmonic and they are the lobes that appear approximately between the sidelobe peaks at the fundamental.

The nonlinear grating lobe phenomenon is illustrated in figure 1. The scaling of the images is reduced to -25 dB in the first harmonic figure and -50 dB in the second harmonic. This has been done to make the grating lobes appear clearer since noise is removed. The lobes in the middle of the second harmonic are the nonlinear grating lobes. The phenomenon resembles fingers. The ratio between sine to the grating lobe and this nonlinear grating lobe is found to be approximately .49 in this example.

VI. Conclusion

In our study we have found that a grating lobe at $\sin \phi_g$ at the fundamental fundamental frequency results in a new secondary grating lobe at $0.5 \cdot \sin \phi_g$ at the second harmonic. An explanation for this phenomenon can probably be found from the theory of two intersecting beams in a nonlinear medium.

Although details have not been given here, we have found that sidelobes have been reduced by a significant amount for the sparse arrays we have considered. These arrays have between 208 and 880 connected elements on the receiver and transmitter out of the total 2500 elements. In general the two-way sidelobe level at the second harmonic has been reduced by 20-25 dB compared to the fundamental.

VII. Acknowledgement

Thanks to Dr. Kai Thomenius of GE Corporate R&D in Schenectady, NY for letting us build on his nonlinear simulation code. Also thanks to Prof. H. Hobæk, Univ. of Bergen for pointing out the similarity between the secondary grating lobes and the "fingers" of nonlinear propagation.

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