Online Control and Near-Optimal Algorithm for Distributed Energy Storage Sharing in Smart Grid

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Abstract—This paper proposes an online control approach for real-time energy management of distributed energy storage (ES) sharing. A new ES sharing scenario is considered, in which the capacities of physical ESs (PESs) are reallocated to users, so that each user manages its own virtual ES (VES) without knowing detailed operations of the PESs. To optimize the ES sharing system in real time, an online algorithm is developed based on Lyapunov optimization framework. The advantage of the online algorithm is that it makes decisions only based on the realization of current system states, without having to predict future uncertain system states such as electricity price, user load, and renewable generation. In performance analysis, it is proven that the online solution is feasible and has a provable performance guarantee. Based on the analysis, an approach for optimal offline parameter selection is proposed to guarantee the online control performance. For practical need of privacy protection, a distributed implementation of the online control is proposed via alternating direction method of multipliers (ADMM). In the distributed implementation, users are allowed to manage their VESs locally without sending their private data to anyone. In simulation, actual real-time data of electricity price, home load, and home renewable generation is used. Results show that the proposed distributed online control approach can provide a near-optimal solution, compared with other benchmarks.

Index Terms—Distributed energy storage sharing, online algorithm, smart grid, virtual energy storage.

I. INTRODUCTION

ENERGY storage (ES) has been considered as a critical technology in smart grid for improving power quality and system reliability [1]. It enjoys a wide range of applications in different levels of power systems. In renewable resource integration, ES reduces variations of intermittent renewable generation and makes renewable energy dispatchable [2]. In distribution networks and microgrids, ES serves as backup generation to achieve dynamic power balance [3]. For end users, ES shifts loads to price valleys, reducing electricity bills [4], [5]. Also, ES can act as distributed energy resources and participate in system-wide peer-to-peer energy trading [6]–[8], achieving energy arbitrage. In spite of these expected benefits, the penetration of ES is not high as far. At present, most of the grid-connected ES is limited to hydroelectric pumped storage, and ES only accounts for about 1.9% of total power generation capacity in China and 2.2% in the United States [9]. One of the major barriers to the widespread use of ES is the high one-time investment cost. To meet this challenge, the concept of ES sharing [10]–[12] has been proposed, in which users do not need to buy ESs for themselves but access the shared ESs according to their demands. Recent studies [12], [13] show that compared with a no-sharing setting where users operate their own ESs separately, ES sharing can bring out lower costs of users using ESs and higher utilization of ESs.

In the existing research on ES sharing, most works focus on a scenario where a single ES is shared by multiple users [13]–[18]. In [14], a price-based method is proposed to allocate the capacity of a shared ES to users. In [15], a negotiation mechanism is designed to further allow users to trade their capacities. It is also feasible to not assign ES capacities to users. In [13], [16], users’ charging/discharging decisions of a shared ES are optimized as a whole, rather than splitting the ES. However, the approaches in [13]–[16] are offline approaches, which assume that future system states in a certain period (e.g., loads and renewable generation in the next 24 hours) are known a priori. The practical value of offline approximation approaches would be low if the state forecasting is inaccurate.

Several studies employ game theoretical approaches in the ES sharing research [17]–[19]. In [17], a privacy aware
stochastic game model is developed, in which users compete for the limited capacity of an ES. In [18], historical home load data is employed to estimate cumulative distribution functions of peak loads, based on which a coalitional game approach is proposed to minimize ES costs. In [19], an auction model is used for setting energy prices, at which sellers and buyers trade energy of ESs in a Stackelberg game. However, sequential charging/discharging constraints of ES are ignored in [17]–[19], which may cause their computational results infeasible to real ES systems.

As presented above, the issue of online ES sharing management together with practical ES models is underexplored in the previous literature. Therefore, this paper proposes an online control approach for real-time energy management of distributed ES sharing. Specifically, we consider that multiple physical ESs (PESs) are shared among multiple users while the users manage their own virtual ESs (VESs) without knowing detailed control of the PESs. In problem formulation, practical charging/discharging models for both PESs and VESs are used. Considering the uncertainty of user load, renewable generation, and electricity price, we develop an online algorithm to optimize the ES sharing system in real time. The online algorithm is designed based on Lyapunov optimization framework [20], [21], and it makes decisions only according to the realization of random system states at current time slots. Further, a distributed implementation of the online control is proposed based on alternating direction method of multipliers (ADMM) [22]. Via the distributed implementation, users and PESs carry out local computation separately without disclosing their private data. In simulation, actual real-time data of home power consumption, renewable generation, and electricity price from Dataport database [23] is used. Results show that the proposed online control approach yields a near-optimal solution, compared with other benchmarks. Also, the distributed implementation demonstrates fast convergence rates and good scalability.

The contributions of this paper are summarized as follows:
• An online control algorithm is developed for real-time energy management of the ES sharing system. It doesn’t need to predict the uncertain system states.
• In performance analysis, we prove that the online solution is feasible and has a provable performance guarantee. Also, we propose an approach for optimal offline parameter selection to ensure the online algorithm’s performance.
• A distributed implementation of the online control is proposed to allow users to manage their VESs locally. Users don’t need to know the details on the PES side and don’t need to send their private data (e.g., renewable generation and grid energy consumption) to anyone.

Regarding the online algorithm design, we employ similar theories to [24]–[26] where ESs are used for mitigating energy imbalance in power networks. Compared with [24]–[26], this paper studies a new ES sharing scenario, which leads to the difference in optimization objectives, constraints, and assumptions. This in turn causes that the proposed algorithm has different results in feasibility and suboptimality analysis and a different method for offline parameter selection.

### II. SYSTEM MODEL AND PROBLEM STATEMENT

#### A. ES Sharing System Model

Fig. 1 shows an ES sharing system where \( M \) PESs are shared by \( N \) users. The capacities of PESs are virtualized and reallocated to users, forming VESs on the user side. We index each user and its VES by \( i \in \{1, \ldots, N\} \) and each PES by \( j \in \{1, \ldots, M\} \). Each user has a photovoltaic (PV) power generation system. Consider discrete-time control of the ES sharing system, and index each time slot by \( t \in \{1, \ldots, T\} \). The center is responsible for the coordination between users and PESs, and for maintaining the system’s energy balance. The center and users can only buy energy from the grid without selling energy back. Advanced communications and computing technologies [27], [28] could implement the data interaction among the entities in Fig. 1. The PESs are already purchased and deployed in the system, and each PES can be owned by itself. We consider that the locations of users and PESs can be different, but all of them are connected with a local power network. We assume that the power line capacities are high enough, so the constraints of power lines in the system are excluded for simplicity. This assumption is often used in the research on consumer-level energy management [11]–[19].

1) User: Let \( s_{i,t} \) denote the energy stored in VES \( i \) at time slot \( t \). Let \( e_{i,t} \) denote the charging/discharging energy. The energy state evolution of VES \( i \) is given by

\[
s_{i,t+1} = s_{i,t} + e_{i,t},
\]

where \( e_{i,t} > 0 \) indicates charging and \( e_{i,t} < 0 \) indicates discharging. \( s_{i,t} \) and \( e_{i,t} \) are respectively bounded by

\[
0 \leq s_{i,t} \leq s_i^{\text{max}},
\]

\[
-e_i^{\text{max}} \leq e_{i,t} \leq e_i^{\text{max}},
\]

where \( s_i^{\text{max}} \) is the VES capacity allocated to user \( i \). Related ES capacity allocation approaches can be found in [14], [18], in which \( s_i^{\text{max}} \) can be selected by users based on their historical load and renewable data. This paper focuses on real-time control of ESs after capacity allocation, so \( s_i^{\text{max}} \) is a known parameter here. Since VESs are virtual devices, we can set zero energy leakage and zero loss of charging/discharging in (1), and set zero minimum energy state in (2). Let \( D_{i,t} \) and \( R_{i,t} \) denote the energy demand and PV generation, respectively, which are random parameters with \( 0 \leq D_{i,t} \leq D_i^{\text{max}} \) and \( 0 \leq R_{i,t} \leq R_i^{\text{max}} \). Let \( g_{i,t} \) and \( h_{i,t} \) represent the energy bought...
from the grid and energy discarded, respectively. The energy balance of user $i$ at time slot $t$ is described by

$$
D_{i,t} - R_{i,t} + e_{i,t} - g_{i,t} + h_{i,t} = 0,
$$

$$
g_{i,t} \geq 0, \quad h_{i,t} \geq 0.
$$

(4)

(5)

2) PES: For PESs, a more practical charging/discharging model [24] is considered. Let $r_{j,t}$ denote the energy stored in PES $j$. Let $c_{j,t}$ and $d_{j,t}$ stand for the charging and discharging energy, respectively. The energy state evolution of PES $j$ is described by

$$
r_{j,t+1} = \mu_j r_{j,t} + \eta_j^c c_{j,t} - \left(\frac{1}{\eta_j^d}\right) d_{j,t},
$$

where $0 < \mu_j \leq 1$ is the leakage coefficient; $0 < \eta_j^c \leq 1$ and $0 < \eta_j^d \leq 1$ are charging and discharging efficiencies, respectively. $r_{j,t}$, $c_{j,t}$, and $d_{j,t}$ are respectively bounded by

$$
r_{j}^{\text{min}} \leq r_{j,t} \leq r_{j}^{\text{max}}, \quad 0 \leq c_{j,t} \leq c_{j}^{\text{max}}, \quad 0 \leq d_{j,t} \leq d_{j}^{\text{max}}.
$$

(6)

(7)

(8)

(9)

3) Center: The center is responsible for keeping energy balance among all VESs and PESs. Let $b_t$ and $a_t$ denote the energy bought and energy discarded by the center. They have

$$
b_t \geq 0, \quad a_t \geq 0.
$$

(10)

The energy balance of the ES sharing system is given by

$$
\sum_{i=1}^{N} e_{i,t} + \sum_{j=1}^{M} (d_{j,t} - c_{j,t}) + b_t - a_t = 0.
$$

(11)

B. Time Average System Cost Minimization

Frequent charge-discharge cycling may accelerate the depreciation of PESs, which incurs operation and maintenance costs of PESs. According to [14], [29], this cost can be estimated by

$$
C_{c,j} c_{j,t} + C_{d,j} d_{j,t},
$$

where $C_{c,j} > 0$ and $C_{d,j} > 0$ are constants. In real situations, the PV system of user $i$ generates only $R_{i,t} + h_{i,t}$ units of energy. These $h_{i,t}$ units are regarded as the amount of solar energy that user $i$ discards. Let $S_i > 0$ be the coefficient to measure the degree that solar energy is wasted on user $i$. For the center, the energy discarded may be consumed by a load or an ES outside the system. Let $S > 0$ be the price of the center using outer energy resources. Let $P_t$ be the energy price of the grid, which is a random parameter with $0 \leq P_t \leq P^{\text{max}}$. Thus, the system cost at $t$ is given by

$$
f_t(x_t) = \sum_{i=1}^{N} \left( P_t g_{i,t} + S_t h_{i,t} \right) + \sum_{j=1}^{M} \left( C_{c,j} c_{j,t} + C_{d,j} d_{j,t} \right) + P_t b_t + S a_t,
$$

(12)

where $x_t = \{e_{i,t}, g_{i,t}, h_{i,t}, c_{j,t}, d_{j,t}, b_t, a_t\}$ is the control decision at $t$; $e_{i,t}$, $g_{i,t}$, and $h_{i,t}$ are decision vectors on the user side; $c_{j,t}$ and $d_{j,t}$ are decision vectors on the PES side. The goal of the ES sharing system is to minimize time-average system cost in the control period, which is described by

$$
\text{P1: } \min_{x_t \in X_t} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[f_t(x_t)],
$$

\text{s.t. (1)–(11)},

where constraints (13) ensure that the system does not discard energy bought from the grid, and constraint (14) indicates that a PES cannot be charged and discharged at the same time. The random system state at $t$ is denoted by $A_t = \{D_t, R_t, P_t\}$ where $D_t$ and $R_t$ are load and PV vectors on the user side, respectively. The expectation in P1 is with respect to the random system states $\{A_t\}_{t=1}^{\infty}$ and possibly random control decisions $\{x_t\}_{t=1}^{\infty}$. Note that $\{x_t\}_{t=1}^{\infty}$ can be random if an algorithm randomly selects $x_t$ in a given decision space at every time slot. We don’t mean that the proposed algorithm will make random decisions.

In P1, the operations of users, PESs, and the center are jointly optimized, which is different from a decentralized setting where users individually make charging/discharging decisions. The proposed setting implies less decision-making independence of users but could lead to a lower system-wide cost. The reason for adopting VESs is to reduce the complexity of ES management. If we allocate a portion of each PES’s capacity to each user, then the system needs to manage $M \times N$ PES-user pairs. In the proposed model, the system only manages $M + N$ ESs, and each user can focus on its own VES management without the need to know the detailed operations of PESs.

III. ONLINE CONTROL ALGORITHM

In real situations, state $A_t$ is highly uncertain and unknown in advance, so it is impractical to attain the optimal solution to P1. In this section, we develop an online algorithm based on Lyapunov optimization framework [20], [21] to provide a near-optimal solution to P1. Lyapunov optimization is a theory that enables time-average constrained optimization in stochastic systems. It can make decisions only according to the realization of $A_t$ at current time slots, so the probability distribution of $A_t$ and forecasting $A_t$ are not required. The proofs of lemmas, theorems, and propositions in this section are presented in Appendices A and C–H.

A. Algorithm Design

1) Lyapunov Drift: For the sake of controlling VESs and PESs, we introduce new parameters $[\alpha, u, \beta, w]$ in Lyapunov optimization framework. These parameters will be precisely selected to make the online solution feasible to P1, which will be discussed in detail in Section III-B. Here, we introduce parameters $\alpha_t$ and $\beta_t$ for VES $i$ and PES $j$, respectively, so that $s_{i,t}$ and $r_{j,t}$ can be shifted to the following versions that the online algorithm can handle:

$$
\tilde{s}_{i,t} = s_{i,t} + \alpha_t,
$$

$$
\tilde{r}_{j,t} = r_{j,t} + \beta_t.
$$

(15)

(16)

Then, we introduce weight parameters $u_i > 0$ and $w_j > 0$, and we define the weighted Lyapunov function [30] as

$$
L(\Theta) = \frac{1}{2} \sum_{i=1}^{N} u_i \tilde{s}_{i,t}^2 + \frac{1}{2} \sum_{j=1}^{M} w_j \tilde{r}_{j,t}^2,
$$

(17)
Algorithm 1: Online Algorithm for Solving P1

1. Offline phase: parameter selection of $\{\alpha, u, \beta, w\}$.

2. Online phase: for $t = 1, 2, \ldots$ do

3. Observe the realization of system state $A_t$.

4. Solve P2 and get the optimal solution $x_t^*$. 

5. $c_{j,t} = \max(e_{j,t}^+ - d_{j,t}^+, 0), \forall j$.

6. $d_{j,t} = \max(d_{j,t}^+, e_{j,t}^+, 0), \forall j$.

7. $\hat{x}_t = [e_{j,t}^+, g_{j,t}^+, h_{j,t}^+, \hat{c}_t, \hat{d}_t, b_t^+, a_t^+]$.

8. Use $\hat{x}_t$ to update $s_{t+1}, r_{j,t+1}$ according to (1), (6).

where $\Theta_t = [\tilde{s}_{1,t}, \ldots, \tilde{s}_{N,t}, \tilde{r}_{1,t}, \ldots, \tilde{r}_{M,t}]$. Equation (17) is used for estimating the overall ES energy state in the system. Define one-slot Lyapunov drift as $\Delta(\Theta_t) = E[L(\Theta_{t+1}) - L(\Theta_t)]$. If $\Delta(\Theta_t)$ is minimized at every time slot, the ESs can be constantly pushed to a lower congestion state, i.e., the ESs are not over charged/discharged.

2) Drift Bound: The upper bound of $\Delta(\Theta_t)$ is given in Lemma 1, where given $\Theta_t$ and fixed $\{\alpha, u, \beta, w\}$, $Z$ and $A$ are unchanged for any control strategy.

Lemma 1: The Lyapunov drift is upper bounded by

$$\Delta(\Theta_t) \leq Z + A + E \left[ \sum_{j=1}^{M} \sum_{i=1}^{N} w_{ji} \hat{r}_{ji,t} \left( \frac{\eta_j^+ c_{j,t} - d_{j,t}^+}{\eta_j^+} \right) + \sum_{i=1}^{N} u_i \hat{s}_{ti,t} e_{i,t} \right],$$

where $Z = \frac{1}{2} \sum_{i=1}^{N} u_i (e_{i,t}^+)^2 + \frac{1}{2} \sum_{i=1}^{M} w_{ij} \eta_j^+ (1 - \mu_j) \beta_j + \sum_{i=1}^{N} w_{ij} \eta_j^+ (1 - \mu_j) \beta_j - d_{j,t}^+(\eta_j^+)^2$, and $A = \sum_{j=1}^{M} w_{ij} ((1 - \mu_j) \beta_j + (1 - \mu_j) \beta_j - d_{j,t}^+(\eta_j^+)^2)$.

3) Drift-Plus-Cost: The slot $t$ problem which minimizes the drift-plus-cost is described by

$$\text{P2:} \min_{x_t} \sum_{i=1}^{N} u_i \hat{s}_{ti,t} e_{i,t} + \sum_{j=1}^{M} w_{ji} \hat{r}_{ji,t} \left( \frac{\eta_j^+ c_{j,t} - d_{j,t}^+}{\eta_j^+} \right) + f_t(x_t),$$

s.t. (3)–(5), (8)–(11).

In P2, we minimize the upper bound of the Lyapunov drift plus one-slot system cost $f_t(x_t)$. Compare with P1, P2 excludes time coupling constraints (1), (2), (6), (7) and nonconvex constraints (13), (14). P2 is a linear programming problem.

Based on P2, an online algorithm is proposed to solve P1, which is shown in Algorithm 1. The control decision produced by the algorithm at $t$ is denoted by $\hat{x}_t$. On line 1, the parameter selection of $\{\alpha, u, \beta, w\}$ in the offline phase will be presented in Section III-D. Lines 5 and 6 aim for making $\hat{x}_t$ feasible to (14) as well as (8), (9). Note that $d_{j,t} - c_{j,t} = d_{j,t}^+ - e_{j,t}^+$ holds, so $\hat{x}_t$ still satisfies (11). Line 7 indicates that $\hat{x}_t$ satisfies (3)–(5), and (10), Line 8 implies that (1) and (6) are met. Next, we will prove that $\hat{x}_t$ also can satisfy (2), (7), and (13), so $\hat{x}_t$ will be completely feasible to P1.

B. Feasibility Analysis

Here, we provide theoretical analysis, showing that Algorithm 1 can meet constraints (2), (7), and (13). The analysis takes advantage of the parameter features of a practical ES sharing system, which is detailed in Appendix B. First, we show that online solution $\hat{x}_t$ satisfies constraint (2) if the parameters of VES $i$ (i.e., $a_i$ and $u_i$) are selected appropriately. In Lemma 2, we characterize the optimal charging/discharging decision of VES $i$ in P2. Based on Lemma 2, we derive Theorem 1 which shows the feasible ranges of $a_i$ and $u_i$.

Lemma 2: The optimal solution to P2, $x_t^*$, has the following properties:

1. $e_{i,t}^* = e_{i,t}^\max$ whenever $s_{i,t} < K^c_i/u_i - a_i$,
2. $e_{i,t}^* = -e_{i,t}^\max$ whenever $s_{i,t} > K^d_i/u_i - a_i$,

where $K^c_i = \alpha_i^\max - S$ and $K^d_i = \alpha_i^\max + S$.

Theorem 1: The VES control sequence produced by Algorithm 1, $\{i_1^\hat{t}, i_2^\hat{t}, \ldots\}$, ensures that energy state $s_{i,t}$ is bounded within $[0, s_{i,t}^\max]$, $\forall t$ if the parameters of VES $i$ satisfy

$$u_i^\min \leq u_i, \quad \alpha_i^\min \leq \alpha_i \leq \alpha_i^\max,$$

where $u_i^\min = (K_i^d - K_i^c)/\alpha_i^\max - 2e_{i,t}^\max$, $a_i^\min = K_i^d/u_i + e_{i,t}^\max - s_{i,t}^\max$, and $a_i^\max = K_i^d/u_i - e_{i,t}^\max$.

Similarly, constraint (7) can be met if we carefully set the parameters of PES $j$ (i.e., $\beta_j$ and $w_j$). Lemma 3 presents important properties of $x_t^*$ with regard to PES control. Using Lemma 3, we can determine the feasible ranges of $\beta_j$ and $w_j$, which are given in Theorem 2.

Lemma 3: The optimal solution to P2, $x_t^*$, has the following properties:

1. $d_{j,t}^* = 0$ whenever $r_{j,t} < B_j^c/w_j - \beta_j$,
2. $e_{j,t}^* = 0$ whenever $r_{j,t} > B_j^d/w_j - \beta_j$.

where $B_j^c = \eta_j^c (C_j^d - \eta_j^c)^2/\mu_j$ and $B_j^d = (S - C_j^d)/\mu_j$.

Theorem 2: The PES control sequences produced by Algorithm 1, $\{i_1^\hat{t}, i_2^\hat{t}, \ldots\}$ and $\{\hat{d}_{j,1}, \hat{d}_{j,2}, \ldots\}$, ensure $r_{j,t}$ is bounded within $[r_{j,t}^{\min}, r_{j,t}^{\max}]$, $\forall t$ if the parameters of PES $j$ satisfy

$$w_j^\min \leq w_j,$$

$$\beta_j^\min \leq \beta_j \leq \beta_j^\max,$$

where $w_j^\min = \mu_j (B_j^d - B_j^c)/\mu_j - (\alpha_j^\min - (\eta_j^c)^2 - \eta_j^c)^2/\mu_j$, and $B_j^d = B_j^d/w_j - (\alpha_j^\min - (\eta_j^c)^2 - \eta_j^c)^2/\mu_j$.

Proposition 1 shows that Algorithm 1 satisfies constraints (13) automatically.

Proposition 1: The online solution $\hat{x}_t$ always meets $\hat{b}_t a_t = 0$ and $\hat{g}_t \hat{h}_t t = 0, \forall t$.

C. Suboptimality Analysis

Here, the suboptimality of Algorithm 1 is analyzed based on the assumption that state $A_t$ is independent and identically distributed (i.i.d.). At first, we characterize the online solution $\hat{x}_t$ with respect to the optimal solution to P2, $x_t^*$, in Lemma 4.

Lemma 4: Under Algorithm 1, the objective value of P2, $\phi_2(\hat{x}_t)$, is bounded by

$$\phi_2(x_t^*) \leq \phi_2(\hat{x}_t) \leq \phi_2(x_t^*) + V,$$

where $V = \sum_{j=1}^{M} w_j^\min \eta_j^c a_j$ and $\eta_j^c = \min\{w_j^\min | (\alpha_j^\min - 1)/\eta_j^c + C_j^d, 0\}$, and $\phi_2^\min = \max\{-(\alpha_j^\min - d_{j,t}^\max) - d_{j,t}^\max\}$. 

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Then, we analyze the following problem, a relaxation of P1:

$$\text{P3: } \min_{\{x_t\}} \lim_{T \to \infty} \frac{1}{T} \mathbb{E}[f_t(x_t)],$$

s.t. (3)--(5), (8)--(11), (13), (14),

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[e_{t,i}] = 0, \quad (24)$$

$$(1 - \mu_j) r^{\mu}_{j} \leq \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\eta^q c_{j,t} - d_{j,t}/\eta^d] \leq (1 - \mu_j) r^{\mu}_{j}.$$

(25)

Compared with P1, P3 relaxes (1) and (2) to (24) and relaxes (6) and (7) to (25). It is shown that any feasible solution to P1 must satisfy P3, so P3 is a relaxation of P1. Letting $\varphi^*_i$ and $\varphi^*_j$ represent the optimal objective values of P1 and P3, respectively, we have $\varphi^*_j \leq \varphi^*_i$. P3 has an important property which is described in Lemma 5. The lemma is derived from [20, Th. 4.5]. The detailed proof is omitted here.

**Lemma 5 (Optimal Stationary Policy [20]):** If system state $A_i$ is i.i.d. over time slots, there exists a stationary solution $x^*_i$ (i.e., the solution is chosen according only to the current realization of $A_i$) that satisfies (3)--(5), (8)--(11), (13), (14), and provides

$$\mathbb{E}[e_{t,i}] = 0, \quad (26)$$

$$(1 - \mu_j) r^{\mu}_{j} \leq \mathbb{E}[\eta^q c_{j,t} - d_{j,t}/\eta^d] \leq (1 - \mu_j) r^{\mu}_{j}, \quad (27)$$

$$\mathbb{E}[f_t(x_t)] = \varphi^*_j. \quad (28)$$

Based on Lemmas 1, 4, and 5, we attain Theorem 3 which characterizes the suboptimality of Algorithm 1.

**Theorem 3:** If system state $A_i$ is i.i.d. over time slots, the time average system cost $\varphi_1$ under the online control sequence $\hat{x} = (\hat{x}_1, \hat{x}_2, \ldots)$ produced by Algorithm 1 is bounded by

$$\varphi^*_i \leq \varphi_1(\hat{x}) \leq \varphi^*_i + Z + V + W,$$

where $W = \sum_{j=1}^{M} \omega_{ji} (1 - \mu_j) \varphi_j$ and $\varphi_j = \max\{r^{\mu}_{j} + \beta^2, (r^{\mu}_{j} + \beta^2)^2\}$.

The general idea for proving the theorem is to get the upper bound of the drift-plus-cost using (18), (23), and (26)--(28). The detailed proof is omitted here. The proof technique can be found in [20], [21], [24]. Theorem 3 means that the online algorithm has a performance guarantee that holds with respect to the worst-case distribution of the i.i.d. system state $A_i$. Also, the algorithm could be analyzed under the assumption of non-i.i.d. $A_i$, for which different forms of suboptimality bounds can be derived. Readers are referred to [20], [26], [31] for more details.

**D. Offline Parameter Selection**

Note that in Theorem 3, the suboptimality bound $V + Z + W$ is a function of $[a, u, \beta, W]$. To improve the online algorithm’s performance, we can minimize the bound $V + Z + W$ subject to (19)--(22) in the offline phase. The bound minimization problem is separable with respect to VESs and PESs since we assign an independent weight parameter to each ES in (17). The suboptimality bound minimization problem for VES $i$ is given by

$$\min_{u_i, \alpha_i} u_i (e^{\mu}_{i})^2, \text{ s.t. (19), (20)}.$$

It is obvious that the optimal solution to the above problem is $u^*_i = u_{i, \text{min}}^{\text{min}}$, then $\alpha^*_i = \alpha^*_{i, \text{min}} = \alpha^*_{i, \text{max}}$. The suboptimality bound minimization problem for PES $j$ is given by

$$\min_{w_j, \beta_j} w_j (\beta_j + 1/\eta^d) \leq \max\{r^{\mu}_{j} + \beta^2, (r^{\mu}_{j} + \beta^2)^2\} \geq 0,$$

s.t. (21), (22).

(30)

Note that $\gamma_j$ and $\eta_j$ are functions of $\beta_j$, as shown in Lemma 1 and Theorem 3. Problem (30) is non-convex. In the following, we develop an algorithm to solve the problem.

At first, we reformulate the problem. Take $\gamma_j$ and $\eta_j$ as variables and relax them as follows:

$$\gamma_j \geq \left(1 - \mu_j\right) \beta_j + \eta_j (r^{\mu}_{j} + \beta^2)^2, \quad (31)$$

$$\eta_j \geq \left(1 - \mu_j\right) \beta_j - d_j^{\text{max}} / \eta_j \geq 0, \quad (32)$$

$$\eta_j \geq r^{\mu}_{j} + \beta_j, \quad \eta_j \geq 0, \quad (33)$$

Using Schur complement [24], (31)--(33) can be equivalently transformed to the following matrix inequalities:

$$X_j^r = \begin{bmatrix} \gamma_j & 1 \\ 1 & 1 \end{bmatrix} \geq 0,$$

$$X_j^d = \begin{bmatrix} \gamma_j & 1 \\ 1 & 1 \end{bmatrix} \geq 0,$$

$$X_j^l = \begin{bmatrix} \eta_j & 1 \\ 1 & 1 \end{bmatrix} \geq 0,$$

where $X_j^r, X_j^d, X_j^l, X_j^R$ are symmetric matrices, and $X \succeq 0$ indicates that $X$ is positive semi-definite. Let $\sigma_j = 1/\omega_j j$, i.e., $0 < \sigma_j \leq 1/\omega_j$. If $\sigma_j$ is fixed, problem (30) can be transformed into the following semi-definite problem:

$$\min_{\gamma_j, \eta_j, \beta_j} \frac{1}{2 \sigma_j} \gamma_j (1 - \mu_j) \beta_j + \frac{1}{\sigma_j} \eta_j (1 - \mu_j) \beta_j,$$

s.t. $X_j^r, X_j^d, X_j^l, X_j^R \succeq 0, (22)$.

(34)

The linear search method is used for solving problem (30), which is shown in Algorithm 2. On line 3, problem (34) can be solved efficiently due to its convexity. Note that running Algorithm 2 only requires local data of PES $j$.

Therefore, the offline parameter selection in Algorithm 1 can be summarized as follows: For VES $i$, set $u_i^* = u_{i, \text{min}}^{\text{min}}$ and $\alpha_i^* = \alpha^*_{i, \text{min}} = \alpha^*_{i, \text{max}}$. For PES $j$, run Algorithm 2 to get the optimal $w_j^*$ and $\beta_j^*$.

**IV. DISTRIBUTED IMPLEMENTATION**

If the center runs Algorithm 1 centrally, it has to aggregate required data from all users and PESs. But, in real situations, users may not be willing to directly reveal their private data, such as demand, PV, and grid energy consumption data.
To deal with the privacy issue, we develop a distributed implementation of the online control in this section. In Algorithm 1, the offline parameter selection and observation of $A_i$ are separable with respect to users and PESs. The remaining issue is how to solve P2 in a distributed manner. Here, we propose a distributed algorithm based on ADMM framework [22], [32], in which each of the users and PESs acts as an agent and each agent carries out local computation without sending its private data to anyone.

Here, we focus on P2 and drop subscript $t$. In P2, (11) is the global constraint coupling all PESs and VESs. The variables in (11) are called coupling variables, and we stack them in vector $x^t$. Note that $x^t$ is part of $x$. Denote (11) as $\delta(x^t) = 0$. Introduce a new vector $y$ as a copy of $x^t$. Use $\Phi_2$ to represent the objective of P2. Thus, P2 can be equivalently transformed to the following problem:

$$
P_4: \min_{x,y} \Phi_2(x) + I(x \in \mathcal{X}) + I(\delta(y) = 0), \text{s.t.} \ x^t = y, \quad (35)$$

where $I(A)$ is an indicator function equal to 0 if $A$ is true and $+\infty$ otherwise; $\mathcal{X}$ is the constraint set for meeting (3)–(5), (8)–(10). Let $\bar{v}$ be the dual variable of constraint (36). The standard ADMM [22] for solving P4 is given by

$$
x^{t+1} = \arg \min_{x \in \mathcal{X}} \left\{ \Phi_2(x) + \left( \frac{\rho}{2} \right) \left\| x^t - x^t + y^t \right\|_2^2 \right\}, \quad (37)$$

$$
y^{t+1} = \arg \min_{y \in \mathcal{Y}} \left\{ \| y - x^{t+1} - y^t \|_2^2 \right\}, \quad (38)$$

$$
v^{t+1} = v^t + \delta(x^{t+1}, y^t), \quad (39)$$

where $\rho$ is a positive constant, and $\nu = (1/\rho)\bar{v}$. Superscript $t$ denotes the number of iterations. In our ES sharing model, (38) has a closed-form solution, given by

$$
y^{t+1} = x^{t+1} + v^t + \tilde{\delta}(x^{t+1}, v^t), \quad (40)$$

where $\tilde{\delta}$ is a function vector. The $n$th entry of $\tilde{\delta}$ is given by

$$
\tilde{\delta}_n = \begin{cases} 
-\delta(x^{t+1})/(N+2M+2), & \text{if } n \in N, \\
\delta(x^{t+1})/(N+2M+2), & \text{otherwise},
\end{cases}
$$

where $N$ denotes the set of index numbers corresponding to variables $[e, d, b]$. Using (40), the ADMM can be simplified to the following form:

$$
x^{t+1} = \arg \min_{x \in \mathcal{X}} \left\{ \Phi_2(x) + \left( \frac{\rho}{2} \right) \left\| x^t - \xi^t \right\|_2^2 \right\}, \quad (41)$$

In (41), we have $\xi^t = x^{t+1} + v^t + \tilde{\delta}(x^{t+1}, v^t)$, where $v^t$ is updated following (42).

Using the fact that (41) is separable with respect to PESs and VESs, we develop Algorithm 3 to solve P2 (i.e., P4) in a distributed fashion. Vector $\xi$ is managed by the center, and $\xi^t$ stands for the entry corresponding to variable $x$. The data exchange among the center, users, and PESs is shown in Fig. 2. As shown in Algorithm 3, the private data of user $i$, $[D_i, R_i, g_i, h_i]$, only appears in the local problem solving, so user $i$ doesn’t need to send its private data to anyone. Also, the center cannot recover the exact $[D_i, R_i, g_i, h_i]$ based on $[e_i, \xi^t]$ because a tremendous number of choices of $[D_i, R_i, g_i, h_i]$ can lead to the same $[e_i, \xi^t]$. For example, $[D_i = 0, R_i = 0, g_i = 0, h_i = 1]$ and $[D_i = 1 + S_i/P, R_i = 0, g_i = S_i/P, h_i = 0]$. Thus, the privacy of users is protected. Due to ADMM framework, Algorithm 3 can achieve a $O(1/\tau)$ convergence rate, shown in Proposition 2. The proposition directly applies the analysis results in [33], so the proof is omitted here.

**Algorithm 2:** Algorithm for Solving Problem (30)

1. Set a small step size $\epsilon$ and a large number $R$. Set $\sigma_j = \epsilon$.
2. While $\sigma_j \leq 1/\omega_{\min}^2$
3. Solve problem (34) and get the optimal solution $\tilde{\beta}_j$.
4. If $q_j/(1/\sigma_j, \tilde{\beta}_j) < R$ then
5. $\sigma_j^* = \sigma_j, \tilde{\beta}_j^* = \tilde{\beta}_j$.
6. $\sigma_j \leftarrow \sigma_j + \epsilon$.

**Algorithm 3:** Distributed Algorithm for Solving P2

1. Set $v^t = 0$ and $\xi^t = 0$.
2. For $\tau = 1, 2, \ldots$
3. The center sends $\xi^t$ to users and PESs.
4. Each user $i$ solves the problem below and sends the optimal solution $e_i^{\tau+1}$ to the center.
5. Each PES $j$ solves the problem below and sends the optimal solution $e_j^{\tau+1}$ to the center.
6. The center solves the problem below, gets the optimal solution $b^{\tau+1}$, $a^{\tau+1}$, and updates $\xi^{\tau+1}$.

**A. Parameter Setting**

In simulation, we use actual real-time data of electricity price, home load, and home PV generation from Dataport [23], which is a big residential energy database. The data is used for
Fig. 3. Real-time data from Dataport database [23] during the week of 01/01/2018 to 01/07/2018. (a) ERCOT electricity price data. (b) Load and PV generation data of a home.

Fig. 4. Time average system cost versus (a) total capacity of VESs, (b) total capacity of PESs, (c) charging/discharging efficiency of PESs, and (d) leakage coefficient of PESs.

B. Benchmarks

The proposed online control approach is compared with the following 5 benchmarks:

- **No Sharing**: Assume that each user has a PES with a capacity of \( s_{i}^{\text{max}} \). Each user individually operates its own PES, and its strategy is to charge the PES before discarding redundant energy and to discharge the PES before consuming grid energy.
- **Greedy**: The greedy algorithm minimizes system cost \( f_i \) at every time slot, which is described by
  \[
  \min_{x_t} f_i(x_t) \quad \text{s.t.} \quad (1) - (11), (13), (14).
  \]
- **Simplified Model**: Some existing Lyapunov-based control approaches consider a simplified PES model with zero energy leakage and zero loss of charging/discharging [4], [21], [30]. The proposed control approach with the simplified PES model is tested.
- **No Offline**: The proposed control approach without Algorithm 2 in the offline phase is tested. Random selection of PES parameters \( \beta_j \) and \( w_j \) will impact simulation results dramatically. Thus, we set \( \beta_j^* + 0.01 \) and \( w_j^* + 0.01 \) to see the impact of the small deviation.
- **Lower Bound**: The lower bound of the optimal cost \( \phi_{1}^{*} \) is \( \phi_{1}(\tilde{x}) - Z - V - W \), given by Theorem 3, which shows that \( \phi_{1}^{*} \) lies between the Proposed curve and Lower bound curve.

C. Performance Comparison

Fig. 4(a) shows time average system costs (i.e., the objective of P1) varying with the total capacity of VESs. With VESs, users can store PV energy to reduce the use of expensive energy from the grid. Thus, system costs generally go down when VESs’ capacities increase. Fig. 4(b) demonstrates similar results. Since users’ charging/discharging operations...
are implemented on the PES side, increasing PESs’ capacities generally reduces system costs. No offline, Simplified, and Proposed perform similarly when PESs’ capacities are small. This is because the PES parameters and models have little impact on system costs at that time. In the simulation, we consider a practical case where the total capacity of VESs is no more than that of PESs. In another case that the total capacity of VESs is higher than that of PESs, the proposed control approach still works, but the resultant system costs will be very high since the system has to increasingly buy and discard energy to make up for the lack of PES capacity. When PESs’ charging/discharging efficiencies are low, the system needs additional energy (from PV or grid) to make up the energy loss. Thus, increasing charg-

ing/discharging efficiency can reduce system costs, as shown in Fig. 4(c). Fig. 4(d) presents similar results. Note that in Fig. 4(d), the distance between Proposed curve and Lower bound curve decreases as the leakage coefficient increases. This is because the increase of \( \mu_j \) leads to the decrease of \( V + Z + W \).

No sharing fails to exploit the diversity of users to reduce the system-wide cost, so it underperforms compared with other ES sharing approaches. Under Greedy, PESs and VESs are exhaustively used, so some of them are often empty. When grid prices are high, empty ESs cannot release energy to support user loads, which causes that the system has to buy the expensive energy from the grid. In contrast, Proposed takes into account Lyapunov drift in the control period, so the energy states of ESs can be maintained at a relatively stable level. This implies that the stable ESs mostly have certain amount of energy to meet user loads when high electricity prices pop up. Thus, Proposed achieves lower system costs than Greedy. In the case of using Simplified, energy leakage and energy loss of charging/discharging are ignored in PES models, which causes that the resulting control decisions may be infeasible for real PES systems. Evidently, Simplified is inferior to Proposed which takes into account a practical PES model. In the offline phase, Algorithm 2 minimizes the suboptimality bound \( V + Z + W \). As shown in Theorem 3, the online solution can get closer to the optimum if \( V + Z + W \) is smaller. Therefore, Proposed with Algorithm 2 always achieves a system cost closer to the lower bound.

D. Convergence of the Distributed Implementation

At last, we evaluate the convergence rate of Algorithm 3. It is considered that convergence is reached if \(|(\phi^2_2 - \phi^2_2)/\phi^2_2| \leq 10^{-5}\) where \( \phi^2_2 \) is the objective value of P2 at the rth iteration of Algorithm 3 and \( \phi^2_2 \) is the optimal value. We increase the total number of ESs, keeping \( N/M = 2 \) unchanged. As shown in Fig. 5, when there are 900 ESs (i.e., 600 VESs and 300 PESs), convergence can be achieved within 12 iterations in average. The fast convergence rate demonstrates that Algorithm 3 is suitable for real-time control. The fitted curve is generated using the method in [34], in which \( R^2 \leq 1 \) denotes the coefficient of determination. The higher \( R^2 \) is, the better the curve describes the data. Fig. 5 shows that the average number of iterations grows nearly logarithmically as the number of ESs increases. This demonstrates the good scalability of Algorithm 3.

VI. CONCLUSION AND FUTURE WORK

This paper considers a new ES sharing scenario where multiple PESs are shared by multiple users while the users only manage their own VESs without knowing the details of PESs. To optimize the ES sharing system in real time, an online control algorithm based on Lyapunov optimization framework is developed. In performance analysis, feasible ranges of VES/PES parameters are derived, and the performance guarantee of the online algorithm is analyzed. To ensure the online control performance, an algorithm is designed to optimize the selection of PES parameters in the offline phase. Further, a distributed implementation of the online control is proposed via ADMM framework. In simulation, actual data of home load, PV, and electricity price is employed. Simulation results show that the proposed online control approach incorporating practical ES models and optimal offline parameter selection yields a near-optimal solution, and the distributed implementation has good scalability.

The proposed online algorithm makes decisions only based on the current system states, while online control with favorable forecasting and learning abilities would potentially have better performances. In future work, we would like to study ES online control in the case that users have partial abilities to predict their loads and to learn the optimal ES control strategies. Moreover, this paper focuses on energy control without designing the pricing rule for the ES sharing service. According to the results, the proposed approach significantly reduces the system cost. In the future, we would like to study how to reward the users/PESs based on their contributions to the system cost reduction.

APPENDIX A

PROOF OF LEMMA 1

Proof: According to (1) and (15), we have \( \tilde{s}_{i,t+1} = \tilde{s}_{i,t}^2 + e_{i,t}^2 + 2\tilde{s}_{i,t}e_{i,t} \leq \tilde{s}_{i,t}^2 + (e_{i,t}^{\text{max}})^2 + 2\tilde{s}_{i,t}e_{i,t} \), which leads to

\[
\frac{1}{2} u_i (\tilde{s}_{i,t+1} - \tilde{s}_{i,t}^2) \leq \frac{1}{2} u_i (e_{i,t}^{\text{max}})^2 + u_\tilde{s}_{i,t} e_{i,t}. \tag{43}
\]

Using \( y_{j,t} \) to denote \( (\eta_j^2 c_j - d_j/t)^2 \) and following (6), (16), we have \( \tilde{f}_{j,t+1} = \mu_j \tilde{f}_{j,t} + (1 - \mu_j) \beta_j + y_{j,t} \). Squaring both sides,
multiplying \( w_j/2 \), and using \( 0 < \mu_j \leq 1 \) yield
\[
\frac{1}{2} w_j \left( \frac{\gamma_j^2}{f_{j,t+1}^2} - \frac{\gamma_j^2}{f_{j,t}^2} \right) \leq \frac{1}{2} w_j \gamma_j + w_j \mu_j \gamma_j \left[ (1 - \mu_j) \beta_j + \gamma_{j,t} \right].
\]

where \( \gamma_j \) is derived from \( e_{j,t} d_{j,t} = 0 \). According to the definition of \( \Delta(\Theta) \), (43) and (44) prove the lemma.

\[ \blacksquare \]

### Appendix B

**System Parameter Specification**

For PESs, the amount of energy leakage is very small in general and can be covered by charging. Thus, we have
\[
\begin{align*}
(1 - \mu_j) r_j^\max &< \eta_j c_j^\max. \\
\end{align*}
\]

Next, we consider that PESs satisfy
\[
C_j^c < S_j, \quad C_j^d < P_{\text{max}}.
\]

Intuitively, if energy discarding cost \( S_j \) is smaller than charging cost \( C_j^c \), the system would always choose to discard users’ energy rather than store the energy. Similarly, if grid price \( P_j \) is always smaller than discharging cost \( C_j^d \), the system would always buy energy from the grid without using PESs. Thus, it is reasonable to consider a practical ES sharing system satisfying (46). Assumption (46) implies that the proposed feasibility analysis cannot directly apply to a scenario where energy can be sold back to the grid at price \( \pi_t \). This is because \( C_j^c < -\pi_t \) cannot be true. For ease of online control, we further set that the VESs and PESs meet
\[
\begin{align*}
s_i^\max &> 2e_i^\max, \\
r_j^\max - r_j^\min &> \eta_j c_j^\max + d_j^\max / \eta_j,
\end{align*}
\]

which indicate that the available capacity of an ES is larger than the range of charging/discharging control. This is practical for most ES systems if we set the length of a time slot to be short enough [24].

### Appendix C

**Proof of Lemma 2**

**Proof:** To show Lemma 2-1), P2 is equivalently transformed to the following problem by replacing \( g_{i,t} \) and \( a_t \) with other variables,
\[
\begin{align*}
\min_{x_t} & \left[ \sum_{j=1}^{M} \left( C_j^c j_{j,t} + C_j^d d_{j,t} + w_j \mu_j \gamma_j \left( \eta_j c_j - d_{j,t} / \eta_j \right) \right) \\
+ & \sum_{i=1}^{N} \left[ u_i s_i, t, e_{i,t} + P_i (D_i, t_t - R_i, t_t + e_{i,t} + h_i, t_t) + S_i h_i, t_t \right] \\
+ & P_j b_l + \left[ \sum_{i=1}^{N} e_{i,t} + \sum_{j=1}^{M} (d_{j,t} - c_j, t_t) + b_l \right], \\
\text{s.t.} \ (3), (5), (8)-(10), \\
e_{i,t} & \geq -D_i, t_t + R_i, t_t - h_i, t_t, \\
e_{i,t} & \geq -\sum_{j \neq i} e_{j,t} - \sum_{j=1}^{M} (d_{j,t} - c_j, t_t) - b_l,
\end{align*}
\]

The problem can be solved by the partitioning method. We fix all variables except \( e_{i,t} \) and then minimize the problem over \( e_{i,t} \). Since the problem is separable, the optimal \( e_{i,t}^* \) can be obtained by solving the following problem:
\[
\begin{align*}
\min_{e_{i,t}} & \{ u_i s_i, t, t - P_i + S_i e_{i,t}, \\
\text{s.t.} \ (3), (49), (50),
\end{align*}
\]

If \( s_{i,t} < K_i / u_i - \alpha_i \) holds, the optimal solution to the above problem is \( e_{i,t}^* = e_{i,t}^\max \), which proves Lemma 2-1).

Similarly, to show Lemma 2-2), we get an equivalent problem of P2 in which \( h_i, t_t \) and \( b_l \) are replaced with other variables. To solve the problem, we fix all variables except \( e_{i,t} \). The optimal \( e_{i,t}^* \) can be obtained by solving the following problem:
\[
\begin{align*}
\min_{e_{i,t}} & \{ u_i s_i, t - S_i - P_i \} e_{i,t}, \\
\text{s.t.} \ (3), \ e_{i,t} \leq -D_i, t_t + R_i, t_t + g_i, t_t + g_i, t_t, \\
& e_{i,t} \leq -\sum_{i \neq j} e_{j,t} - \sum_{j=1}^{M} (d_{j,t} - c_j, t_t) + a_t.
\end{align*}
\]

If \( s_{i,t} > K_i / u_i - \alpha_i \) holds, the optimal solution to the above problem is \( e_{i,t}^* = -e_{i,t}^\max \), which proves Lemma 2-2).

\[ \blacksquare \]

### Appendix D

**Proof of Theorem 1**

**Proof:** Algorithm 1 has \( \hat{e}_t = e_t^* \), so we prove that \( e_t^* \) can satisfy \( s_{i,t} \in [0, s_{i,t}^\max] \) in the following. The theorem is proved by using mathematical induction. Consider that VES \( i \) has \( s_{i,t} \in [0, s_{i,t}^\max] \) at \( t = 1 \). We need to prove that \( s_{i,t+1} \) will be bounded in \( [0, s_{i,t}^\max] \) when \( s_{i,t} \) has been bounded. Three cases of \( s_{i,t} \) need to be discussed:

1. \( s_{i,t} \in [0, K_i / u_i - \alpha_i] \): Based on Lemma 2-1), we have \( s_{i,t+1} = s_{i,t} + e_{i,t}^\max \geq 0 \). We also have \( s_{i,t+1} < K_i / u_i - \alpha_i \min \max + e_{i,t}^\max \leq 2e_{i,t}^\max < s_{i,t}^\max \), where the second inequality is derived by (19), and the last inequality follows (47).
2. \( s_{i,t} \in [K_i / u_i - \alpha_i, K_i / u_i - \alpha_i] \): In this case, using (20), we can derive \( s_{i,t+1} = s_{i,t} - e_{i,t}^\max \leq K_i / u_i - \alpha_i - e_{i,t}^\max + e_{i,t}^\max = s_{i,t}^\max \).
3. \( s_{i,t} \in (K_i / u_i - \alpha_i, s_{i,t}^\max] \): From Lemma 2-2), we have \( s_{i,t+1} = s_{i,t} - e_{i,t}^\max \leq s_{i,t}^\max \). We also have \( s_{i,t+1} > K_i / u_i - \alpha_i - e_{i,t}^\max > 0 \), which is derived from the fact that \( u_i > 0 \).

\[ \blacksquare \]

### Appendix E

**Proof of Lemma 3**

**Proof:** The proof is similar to that of Lemma 2, we first transform P2 into an equivalent problem in which \( b_l \) is excluded. Then, we fix all variables except \( d_{j,t} \). The optimal \( d_{j,t}^* \) can be obtained by solving the following problem:
\[
\begin{align*}
\min_{d_{j,t}} & \{ -w_j \mu_j \gamma_j \eta_j / \eta_j^2 - C_j^d - P_j \} d_{j,t}, \\
\text{s.t.} \ (9), \ d_{j,t} \leq -\sum_{i=1}^{N} e_{i,t} - \sum_{i \neq j} (d_{j,t} - c_j, t_t) + c_j, t_t + a_t.
\end{align*}
\]

If \( r_j, t_t < B_j^c / w_j - \beta_j \), the optimal solution to the above problem is \( d_{j,t}^* = 0 \), which proves Lemma 3-1).

To show Lemma 3-2), we get an equivalent problem of P2 in which \( a_t \) is excluded, and then fix all variables except
The optimal $c_{i,t}$ can be obtained by solving the following problem:

$$\min_{c_{i,t}} \left( \sum_{j=1}^{N} w_j t_j \tilde{r}_j^2 + C_j^S - \tilde{S}_i c_{i,t} \right)$$

s.t. (8), $c_{i,t} \leq \sum_{j=1}^{N} d_j c_{i,t} + d_{j,t} + b_t$.

If $r_{i,t} > B_{t,j}/w_j - \beta_j$, the optimal solution to the above problem is $c_{i,t}^* = 0$, which proves Lemma 3-2).

**APPENDIX F**

**PROOF OF THEOREM 2**

*Proof:* We first prove that $x_t^*$ can meet (7). Consider that (7) holds at $t = 1$. By mathematical induction, we are going to prove $r_{i,t+1} \in [r_{i,t}^{\min}, r_{i,t}^{\max}]$ when $r_{i,t} \in [r_{i,t}^{\min}, r_{i,t}^{\max}]$ is given. Discuss the following three cases of $r_{i,t}$:

1) $r_{i,t} \in \left[ r_{i,t}^{\min}, B_{t,j}/w_j - \beta_j \right)$: Using Lemma 3-1, we have $r_{i,t+1} = \mu r_{i,t} + \eta^t_{i,j} c_{i,t} \max > \mu r_{i,t} + (1 - \mu) r_{i,t}^{\min} \geq r_{i,t}^{\min}$, where the first inequality is derived from (45). We also have $r_{i,t+1} < \mu r_{i,t} + \eta^t_{i,j} c_{i,t} \max < r_{i,t}^{\max}$, which is obtained by the fact of (46).

2) $r_{i,t} \in \left[ B_{t,j}/w_j - \beta_j, B_{t,j}/w_j - \beta_j \right)$: According to (22), we have $r_{i,t+1} \geq \mu B_{t,j}/w_j - \mu r_{i,t}^{\max} - \eta^t_{i,j} d_{i,t} \max \geq r_{i,t}^{\min}$. And $r_{i,t+1} = \mu B_{t,j}/w_j - \mu r_{i,t}^{\max} + \eta^t_{i,j} c_{i,t} \max = r_{i,t}^{\max}$.

3) $r_{i,t} \in \left( B_{t,j}/w_j - \beta_j, B_{t,j}/w_j - \beta_j \right)$: From Lemma 3-2, we have $r_{i,t+1} = \mu r_{i,t} + \eta^t_{i,j} c_{i,t} \max - d_{i,t} \max \geq r_{i,t}^{\min}$. Based on the fact of (46), we also have $r_{i,t+1} = \mu B_{t,j}/w_j - \mu r_{i,t}^{\max} = r_{i,t}^{\max}$.

The above three points show that $\{c_{i,t}, d_{i,t}\}$ satisfies (7). In Algorithm 1, we have $\hat{c}_{i,t} \leq c_{i,t}^*$ and $d_{i,t} \leq d_{i,t}^*$, which completes the proof.

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